

## Physics and Advanced Technique

On November 7, 1940, a 42-mile an hour wind swept through the Tacoma Narrows Sound and past the newly built Tacoma Narrows suspension bridge. Within minutes the bridge had begun a violent pattern of vibration that rocked the single car on the center portion of the bridge and continued for an hour until the steel and asphalt structure could no longer bear the strain and the center section of the bridge collapsed, sending the abandoned vehicle into the sound. The bridge had only lasted four months. What had caused the 6 million dollar, carefully engineered bridge to collapse after so little strain? The answer: resonance (Knill, 1).

This is a paper about physics: physics and the violin. Although a physicist or violin luthier may indeed enjoy reading this paper for its practical application of what are fundamentally mathematical problems, I'm afraid the paper will contain disappointingly few equations since I have not written a mathematical approach to acoustical physics, nor a guide to making an excellent string instrument. Rather, I plan to present an application of the basic principles of physics to violin playing and technique in an effort to facilitate greater ease of performance. In other words, if you are a violinist, and like me, struggle with a difficult passage or with questions of how to improve after becoming a professional, this paper is for you. It is not a revolutionary approach to technique, nor is it an attempt to analyze and dismember what is already working. It is an approach to the basic physics of the violin and how these principles change and influence advanced technique.

## **Energy: Laws of Physics**

“It is only at the end (if at all) that [musicians] might ask where physics impinges on what they already know.” –Ian Johnston (Johnston, 1)

The ancient Greek philosophers thought a great deal about music. They thought it worthy of their study: mathematically intriguing, and mystically connected to the functions of the universe (Johnston, 9). After the enlightenment however, music began to be viewed solely in an artistic light and soon there were few mathematicians or physicists who devoted their careers to the study of what has come to be called acoustical physics (Johnston, 106). Until the studies of Helmholtz during the years of 1858-1894, very little research existed on the physics behind sound, though a great deal of research already existed on basic physical laws (Johnston, 224). For this reason our study will also begin with basic laws of physics and energy.

We will begin with a definition of a violin in physical terms. In the strictest terms of physics, a violin is a machine. A machine changes one kind of energy into another kind of energy. In our case, the violin changes mechanical energy into acoustic energy (Peterlongo, 71).

All physical objects have mass. Measurement of mass usually occurs in kilograms/grams, so mass is frequently taken by the general public as another term for weight (Johnston, 57). Two important and fundamental laws of motion exist for our consideration. Isaac Newton said that a force is required to start an object in motion. (Johnston, 57). Galileo discovered that “A moving body will continue to move in a straight line unless acted upon by some external force.” These are

commonly called the laws of inertia (Johnston, 57). (I must note here, since I am speaking to musicians, that using the term “force” is less than aesthetically desirable, but since it is the commonly used term in physics, I will continue to use it here.) The most important fact for a violinist to remember about mass is that each string has a different mass from all the others.

We have begun with two of the most basic laws of physics, which state that force is required both to place an object in motion and to change its type of motion. Let us add the basic concept of energy. For this we require an additional law of physics, the law of the conservation of energy: “Energy cannot be created or destroyed. It can only be changed from one form into another.” (Johnston, 59-60). Three types of energy contribute to mechanical energy: potential energy, kinetic energy and entropic energy (heat energy).

Kinetic energy is the energy in a moving object, and equals half the mass of an object multiplied by the velocity of its movement squared. [Kinetic energy =  $0.5(\text{mass}) \times (\text{velocity})^2$ ]. Potential energy is the amount of energy in an object that is unused. So, in simple terms, kinetic energy is moving energy and potential energy is “stored” energy (Johnston, 59). As an example of the interplay of kinetic and potential energy, imagine a bouncing ball under the force of gravity. As I lift the ball upwards, I give it potential energy, but as I release it and it drops through the air, all of the potential energy changes to kinetic energy. As the ball bounces upwards and slows down at the top of its arc, the kinetic energy turns back into potential energy. However, the system is not perfect, because eventually the ball will stop bouncing. The energy that apparently dissipates actually turns into the third type of energy,

heat energy. In a perfect machine, energy is never lost to heat (Johnston, 59), but there are no perfect machines. Rather, there is the closed system of the universe, in which all energy simply changes forms *ad infinitum*. Pendulum clocks come close to being a perfect (Johnston, 60), but even in pendulum clocks, energy seeps out through friction as heat into the air, just as in the case of a bouncing ball. Eventually the pendulum will stop (Johnston, 61). Vibrating systems that bounce (like the ball, or the violin bow) tend to bounce more quickly as the vibrations and energy dissipate. In the case of the kinetic energy of vibrating strings, the energy leaves the string as it transfers through the bridge and the violin body and then comes into contact with the air, where it becomes acoustic energy. In the process of all of these transfers to different objects and mediums, some energy is lost. Most of this lost energy occurs as a result of friction. (The bridge reflects some energy that does not transfer into sound back into the string.)

The small remaining amount of energy is lost because of an imperfect transfer of energy between different materials. This imperfect transfer of energy is usually called impedance; this is the final concept to introduce in this section about energy. Here, I am simply talking about impedance in its simplest form, which is a type of resistance. Therefore, to avoid confusion, I will simply call it resistance.

In the case of strings, two physical factors are in play: mass (due to the tendency of objects to persist in their current state of motion or rest) and elasticity (Peterlongo, 32). Elastic resistance is a value that designates how much stretch an object has (Johnston, 226). Elastic resistance—which can be understood as a type of stored, or potential energy—becomes particularly important in the case of stretched



strings, because each string has different amounts of elasticity and different mass (discussed previously) causing the resistance of each string to differ.

Since energy in the form of acoustic energy and heat continually seeps out of the system (violin) in question, as long as the small amount of energy that has been lost is continually put in again, the system continues in a somewhat stable form of kinetic energy (very much like pushing a child on a swing). In certain objects such as the Tacoma Narrows Bridge, at a certain frequency, the system oscillates rather disproportionately compared to the small amount of force being applied. This is called resonance and it describes the periodic continual exchange between potential and kinetic energy directly responsible for the collapse of the bridge as well as the significantly less catastrophic motion of a violin string (Johnston, 226).

Thus, we find that the resonant frequency of a string is determined by its elasticity and mass. A tighter string will yield a higher frequency (less elasticity) and a looser one will yield a lower frequency (more elasticity). A thicker string (more mass) will yield a lower frequency, and a thinner string (less mass) will yield a higher frequency.

We know that the G string has greater mass than the E string, but it also has greater elasticity. Inertial resistance (mass) makes it difficult to start the vibration, but elasticity increases the energy that is released when vibration is begun. Thus, the two factors are able to cancel each other out in a way. The E string with its smaller inertial resistance and smaller elasticity is easiest to start in motion, but motion must be maintained since the smaller elasticity will not lend itself to a great deal of stored energy. The G string (with larger inertial resistance and larger

elasticity) is more difficult to excite into motion, but maintains a state of motion more easily. The D and A strings, which are less extreme in both types of resistance, are very easy to get in motion, but since they lack some of the energy of the G string and the high tension of the E string, projection becomes more difficult. Some of this can be due to orchestration problems, but a portion is also due to resistance and the way it contributes to sound. Since mechanical resistance is made greater by both mass and greater frequency, it seems that at the extreme ranges of the violin, the violinist must work hardest, but the most sound results. The middle ranges are generally easier to make “speak, but in return, they lose a bit of projection (Johnston, 225).

We see then, that force is required to change an object’s kinetic energy, and this force is subject to a certain impedance or resistance based on the object in question. When impedance of the object is close to zero, it takes only a small force to bring about a huge response (the afore-mentioned resonance). The primary problem of the violin lies in the difficulty first of exciting a string with mass and elasticity into motion—or a kinetic state of energy—and then changing the mode of that string’s kinetic energy at a speed which defies imagination, all while compensating losses of energy in the system.

I would now like to continue on to the specific physics of the violin. I have excluded a great deal of information which relates to the construction of violins because this paper is written for performers and teachers.

Wendy Case 1/25/14 3:57 PM

**Comment:** This is correct.- Andy has looked at it.

## An Overview of Violin Physics

In spite of recent advances in science, many of the subtler points of the physics of the violin remain a mystery (Johnston, 109). However, most of that which relates directly to the player has not only been discovered, but is straightforward enough that it can be understood by a person with little background in physics.

Let us start with a few definitions. (This section will probably contain a number of new terms to most of my readers, but bear with me; each term carries importance.) These terms relate directly to the nature of the vibration of stretched strings.

- *Period* is the time it takes a string to make one complete cycle—one complete “swing,” from one side to the other and back again to the starting point. (Ex. The period of the open A =  $1/440$  sec).
- *Frequency* relates to the pitch of the note and is controlled by the number of complete cycles (“swings”) per second. The number that designates the frequency is equal to the reciprocal of the period and is usually given in Hertz, or vibrations per second [Ex. The frequency of the open A = 440 Hz or 440 vibrations per second (Johnston, 29)(Peterlongo, 31-32)]. Frequency is dependent on the length, line density, and the tension placed on the string. [Frequency =  $[0.5(\text{Length})] \times \text{square root tension/line density}$  (Johnston, 65)].
- *Amplitude* refers to the amount of displacement to either side of equilibrium (rest position) in a vibrating object, in this case, our string. Amplitude relates to volume (Johnston, 29)(Peterlongo, 31).
- *Nodes* are places in the vibrating string which do not move (or zero amplitude).
- *Antinodes* are places in the vibrating string which have the greatest displacement (or amplitude).
- The *fundamental* is the lowest frequency with which a stretched string vibrates. (A4 is the fundamental pitch of the open A string. If the violinist places one finger down, thus shortening the string, the

fundamental pitch becomes B.) (The fundamental is also the first partial)

- *Modes of vibration* is a term that designates the basic patterns (including the fundamental and other partials) of vibrations in a stretched string (Johnston, 44). These are roughly the same whether in pizzicato or arco. The only difference in a bowed string lies in the effect of friction, which will be discussed later. For now, we will refer to a plucked string for simplicity's sake.

When a string vibrates, it moves from side-to-side at the fundamental frequency related to its length, tension, and linear density. Greater length or less tension lowers frequency; shorter length or more tension raises the frequency. Linear density (caused by different core materials in the strings) and mass also matter, but since we violinists do not make our own strings, we will not make this a concern in this paper (Peterlongo, 34). As previously stated, the side to side motion of the A string leaves and returns to its starting place 440 times per second. With the G string, at only 196 vibrations per second, you can clearly see the vibration happening. It looks like a jump rope to the naked eye. Although something slightly more intricate is happening, let us start with the picture in Diagram 1 (Johnston, 31). This represents the early theory of the vibration of strings and since it is visible, you can understand why it gained credibility (Beament, 11).

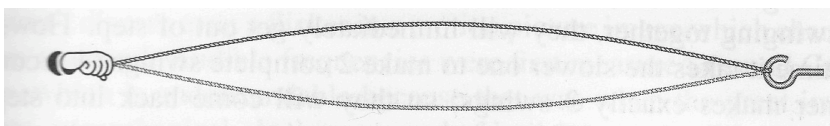


Diagram 1 (Johnston, 31).

It looks as though the string in our diagram involves only one frequency. The actual vibrations are not this simple. As the string vibrates, it moves in several different fashions, not visible to the unaided eye. The first way it vibrates is in terms of the fundamental, and this is where we see the first shape that looks like a jump rope. But the string also vibrates in exact ratios of the original pitch. The string not only vibrates in its whole length, but also in halves, thirds, quarters, fifths, sixths, etc., and the pitches corresponding to these lengths of the string also ring at the same time as the fundamental ( $f_1$ ). So, in any real musical note, there is really energy at multiple frequencies (Johnston, 44). These multiple frequencies are called *partials*. The fundamental (what we generally hear as the sounding pitch) is the first partial; the frequency an octave above that—with a ratio of 1:2, or half of the string—is the second partial, and so on. Diagram 2 shows the second, third, and fourth partial (Johnston, 44).<sup>1</sup>

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<sup>1</sup> See discussion of partials on pg. 3.

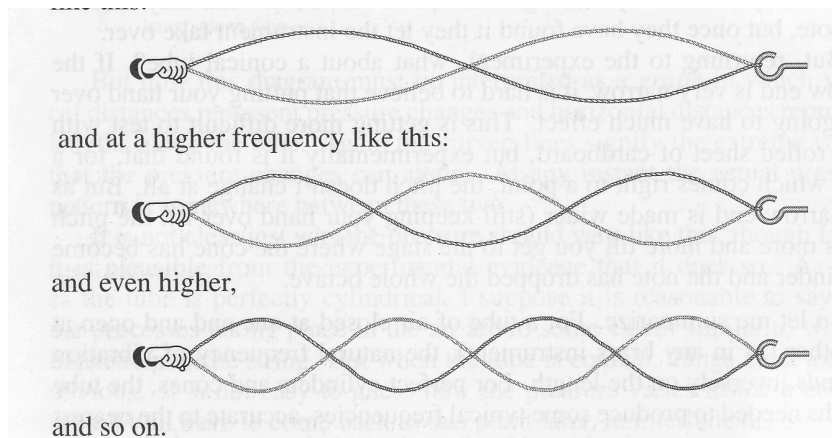


Diagram 2 (Johnston, 44).  $fx_2$ ,  $fx_3$ ,  $fx_4$ .

The arcs were originally thought to be super imposed on each other, as in Diagram 3.

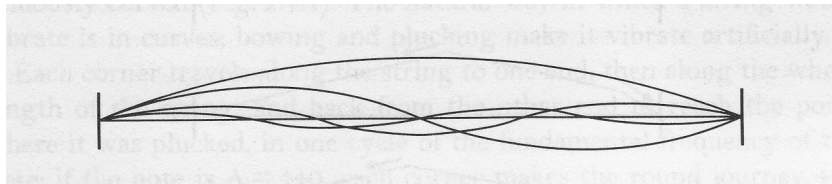


Diagram 3 (Beament, 11).

Of course, an actual picture of string would have looked like a single strange and wobbly line that was really a combination of all of these vibrating modes added together.

However, although these diagrams are helpful in understanding basic string vibration, recent science has added another clarification. What you cannot see is

that, when the finger (or bow) pulls the string sideways, it creates a corner (or notch) in the string. As the string is released, this corner splits into two corners and races up and down the length of the string. As the corner is reflected (just like waves in a tub) from the bridge and the nut, the amplitude decreases and inertia gradually rounds out the corner. The frequency of the fundamental controls how fast the corner moves. On the G string, the visible curve you see is actually only the path of the corner as it runs up and down the string, moving so fast that you only see its circumscribed arc. This is why the first models of string vibration looked like jump ropes. It can help to study the above diagrams, but it is important also to recognize their basic flaws, since the corners of the original diagrams are rounded and thus only an approximation. Diagram 4 is a more accurate representation of what is really happening (Beament, 12).

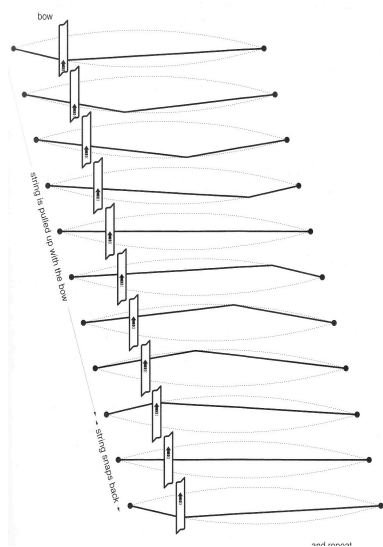


Diagram 4 (Johnston, 121).

This somewhat surprising motion occurs because the string is under tension when the finger pulls it sideways. The shortest distance between two points is a straight line, and this is the distance the corner travels when the finger is released. (Beament, 13). The line segments between the corners act as though they were shorter strings, vibrating in their own manner (Cremer, 41). Over time, as the amplitude of the vibration decreases, frequency of vibration and period do not decrease (Johnston, 30). However, the corners gradually round off due to inertia (Beament, 13). When this happens, the upper partials also die away.

You can see that each portion of the string vibrates in its own unique way and the motions are not only specific to the parts of the string, but also rather complicated. The addition of all of the partials makes the motion even more complex (Johnston, 123). Fortunately, the only part of the string controlled by the player is that portion directly in contact with either the finger in pizzicato or the bow in arco. This is the only portion we need to consider as players.

Let us take a more in-depth look at the partials in a stretched string. It is important to remember that the first partial is, in fact, the fundamental itself ( $f \times 1$ ). The second partial ( $f \times 2$ ) corresponds to the pitch related to half of the string length or period, or the octave above the fundamental (Peterlongo, 32). This constellation of frequencies is called the harmonic series and is shown in Figure 5 (Johnston, 97-98). Note that "harmonic" in this diagram is synonymous with "partial".



| COMPONENT     | FREQUENCY | RATIO | INTERVAL       |
|---------------|-----------|-------|----------------|
| Fundamental   | $f$       |       |                |
| 2nd harmonic  | $2f$      | 2/1   | octave         |
| 3rd harmonic  | $3f$      | 3/2   | perfect fifth  |
| 4th harmonic  | $4f$      | 4/3   | perfect fourth |
| 5th harmonic  | $5f$      | 5/4   | major third    |
| 6th harmonic  | $6f$      | 6/5   | minor third    |
| 7th harmonic  | $7f$      | 7/6   | (3- semitones) |
| 8th harmonic  | $8f$      | 8/7   | (2+ semitones) |
| 9th harmonic  | $9f$      | 9/8   | major tone     |
| 10th harmonic | $10f$     | 10/9  | minor tone     |
|               |           |       | etc. . .       |

Diagram 5 (Johnston, 98)

One of the beauties of string instruments is that at full volume, a single note can contain 20 or more partials. Partial s get higher and closer together as their number increases. The top partials are nearly imperceptible to the ear, and vibrate with very small amplitudes, so the lower partials have the greatest impact on the sound (Beament, 99). The bridge, violin body, and air inside the violin involve themselves in emphasizing or reducing the amplitudes of different partials and the resulting sound spectrum creates a unique sound quality and timbre for every note (Johnston, 124).

In order to proceed further with the harmonic series (the partials in order), we must look at a diagram of what is called the waveform. All sounds can be represented by a graph of pressure over time, and to represent it, we draw something that looks like a picture of ripples on a pond. In a sound with no partials, or a sine tone (which can only be generated electronically), we represent the pressure over time by a perfect sine curve (Peterlongo, 31). This is the wave form of the first partial (fundamental) with the second and third partials below it.

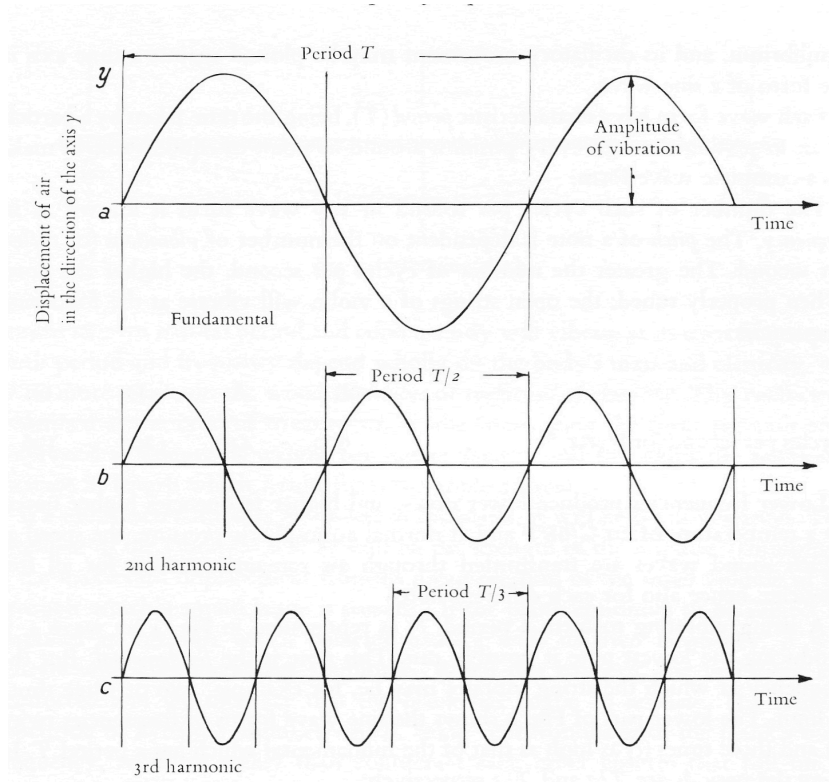
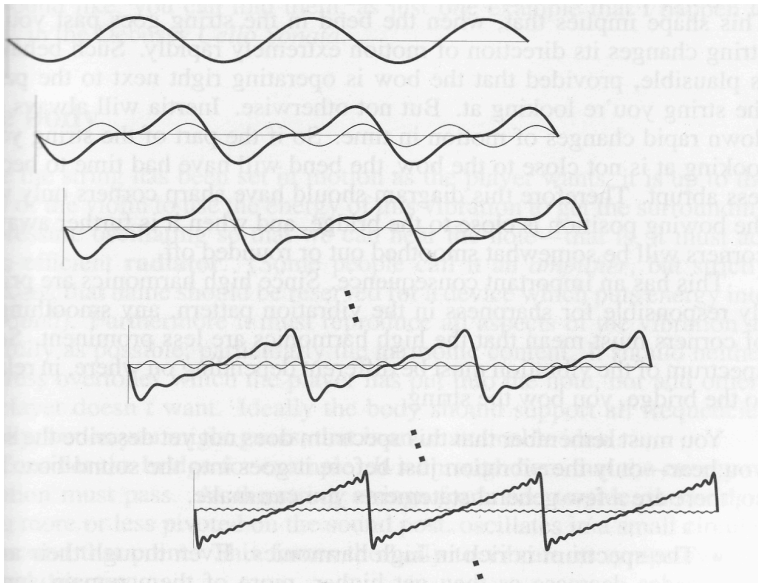


Diagram 6 (Peterlongo, 31).

Notice that this curve looks exactly like our first drawing of the string with fundamental if we were to extend the string beyond the bridge. Of course, while the vibrating fundamental of the string looks like half of a sine curve, the vibration in the air does not actually look like a sine curve. Rather, the sine curve drawing is a representation of the pressure variations that happen in the air as a result of the wave.<sup>2</sup>

When we add further partials to the fundamental, each one makes a slight correction to the original shape of the wave—bringing it closer to the waveform produced by the actual vibration of the string—but not enough to change the wavelength of the fundamental, only the original shape of the sine-curve (Johnston, 125).



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<sup>2</sup> The wave is actually a longitudinal wave comprising compressions and rarefactions in the air.

Diagram 7 (Johnston, 125).

Notice that the final diagram this looks very similar to the actual physical motions of the string with the corner produced by the finger or bow. Since we know that the actual sound of a violin contains at least the first six partials, and that a single tone without any additional partials (generated on a computer) produces a perfect sine wave, perhaps the most important conclusion we can draw at this point is that the harmonic content is directly related to the physical motions of the string (Johnston, 123). A sharper corner of the string indicates a higher frequency content of the composite wave because there are more (and thus higher) partials.

The most influential section of the string is the small area right next to the bridge, because that portion of the string transfers its vibration pattern to the bridge. The bridge in turn, transfers the vibrations to the body of the violin, which transfers them to the air, allowing them to escape from the sound holes (Johnston, 123), as well as causing the violin to vibrate. The bridge acts as a filter of the frequencies produced by the vibrating string. In other words, not all of the frequencies come through in full or equal strength. This happens in part because of the bridge's own resonant frequency (Cremer, 212), and in part because of the position of the axis of vibration of each string in the bridge (Peterlongo, 75). Bridges resonate between 3000 and 6000 Hz. (Cremer, 212)(Beament, 88). As you can imagine, this greatly emphasizes both the high fundamentals (upper E string) and the partials that fall in this range. The range is above the violin's most commonly used fundamental pitches, but not out of the range of the most common partials. If it were not for the curves in the bridge that allow some freedom of movement and

thus provide additional amplitude for the lower partials, the sound produced would emphasize only the upper partials and be extremely strident (Peterlongo, 73).

How does the bridge contribute to sound? The motions of the strings are periodic, and excite periodic motion in the bridge itself (Cremer, 92). Since the pressure on the bridge exerted by different strings is unequal, the bridge forms an asymmetrical vibrating object and filters out different partials on each string. The periodic motion of the bridge is extremely complex and somewhat unstable in regards to its location (Cremer, 206). It centers around an axis of vibration, which constantly changes depending on the string, bow force, and frequency (Peterlongo, 75). The constant changes take extra energy out of the string, but as Cremer says “Losses are necessary at the bridge if sound is to be produced.” (Cremer, 69). The axis (note that it is almost never at the center of the bridge) causes the bridge to rock back and forth from side to side (Cremer, 219). If you review the section on frequency, you will note that the string comes to one side and back again for each period. So does the bridge, but the difference is that the bridge “stomps” one of its feet on the body of the violin on each side as it vibrates back and forth. If you imagine the string swinging to the right with the right-hand bridge foot hitting the violin, then the string swinging to the left and the left-hand bridge foot hitting the violin, you will realize that for one complete period of the string, the bridge will hit the violin twice, once on the right foot, and once on the left foot. Thus, the frequency of vibration created by the bridge on the violin is actually double the frequency of the string. In other words, the bridge emphasizes the second partial. In fact, the bridge is actually a frequency doubler of all frequencies present in a sound. Perhaps

the most convenient result of this phenomenon is that, when playing octaves, if the lower string is more strongly bowed than the upper one, the upper pitch will often not need to be absolutely exact because the double vibration in the bridge will compensate for slight variations by coaxing the upper string into sympathetic compliance (Johnston, 126).

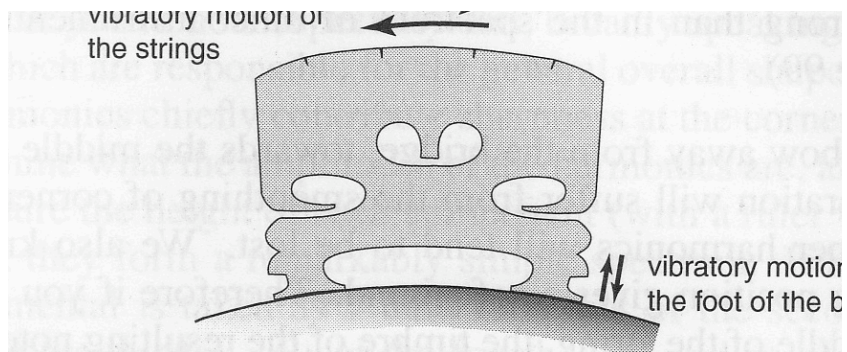


Diagram 8 (Johnston, 126).

Since the axis of vibration is seldom central (i.e. in the exact middle of the bridge), one foot of the bridge hits the body of the instrument harder than the other. If the player plays higher notes, this corresponds to the right (sound post) side of bridge, and the right-hand bridge foot pounds on the right side of the instrument, bringing the that side into the greatest amplitude of vibration. If the notes are lower, the bridge foot on the left (bass bar side) hits the front of the instrument with more force, bringing the left half of the body into the greatest vibration (Peterlongo, 69)(Cremer, 206).

As the bridge transfers its vibrations to the front of the instrument, the violin body begins its function as not only a frequency filter—emphasizing certain partials and dampening others—but also as a resonator (Peterlongo, 68) (Cremer, 240). The amount of energy that transfers from the bridge to the body at a certain pitch depends on both resistance (impedance) and the body of the violin’s ability to resonate at that frequency and its partials (Beament, 73). Violin luthiers tune the plates of the instrument to match certain frequencies by paring away the wood until the right sound is achieved. For example, the violin front is tuned to “D” before the addition of the sound post and bass bar (Beament, 71). Since the violin body is what is called an “indeterminate structure” (too complex and individual to be analyzed without the use of a computer, even for today’s best physicists), we will stay with a few general and helpful observations (Peterlongo, 71).

Wood has elasticity and mass. This means that it can be “deformed” to achieve vibration (Beament, 2). Just as there are nodes in the vibration of a string, there are also nodes in the vibrations of a three dimensional surfaces such as the body of the violin (Johnston, 250-251). This is why the placement of the sound post is important. It must be placed at a node—or a place where the body would not normally vibrate—so as not to interfere with amplitude of vibration (Johnston, 252). The bridge is set very close to this node, so that it’s small motions will have the greatest effect in increasing amplitude in the front of the violin as the waves move outward. (Avoiding incorrect placement of the bridge is also crucial so that sound will not be dampened). The other side of the bridge is set next to the bass bar and excites it into vibration as well. The vibrations on the lower notes actually have

greater amplitude because the bass bar is not anchored to the back (Peterlongo, 75). As we will see during the future discussion on amplitude and intensity, this helps to equalize the sound throughout the range of the instrument since lower notes naturally have less intensity. Optimal placement of the sound post and the bridge will not greatly increase the quality of a poor instrument, but poor placement will destroy the projection of a fine violin (Beament, 50).

The different parts of the belly vibrate in opposite directions. In other words, when the right side goes up, the left goes down and vice versa (Peterlongo, 69-70). The bass bar continues to excite the front of the instrument, and the sound post and ribs transfer these vibrations to the back (Cremer, 206), which reflect most of them because of the greater stiffness and thicker plate of the wood (Peterlongo, 50). (Maple, usually the wood for the back, is harder and denser than the traditional and flexible pine front.). The body of the violin acts as an efficient coupler (transfer point) of energy as the waves spread out on the surface of the wood, because more air comes into contact with the vibrations (due to the fact that the violin body has larger surface area than the strings) (Johnston, 178). The flexing of the wood of the violin produces air pressure differences on different sides inside the violin. This excites the air into vibration as it is continually compressed and expanded and escapes through the sound holes (Johnston, 126) (Peterlongo, 67-70). The sound holes reduce mass for the bridge to move, and thus determine the type of vibration in the front of the violin. Since sound holes also allow for freedom of motion in the air (Peterlongo, 51), they form the biggest single determining factor in the overall sound of a violin. Every maker uses his own signature style. Stradivarius and



Guarneri del Gesu themselves had very different sizes of sound holes and this accounts for a large portion of the difference in the sounds of these very fine instruments (Peterlongo, 51).

Two more small items of importance to players must be addressed. Both deal with the area of the strings behind the bridge and reinforce the notion that a bridge set-up by a professional luthier is crucial. Restorers are usually very good at arranging set-up, since they both understand the violin-making craft and care deeply about its history. The string length behind the bridge is extremely important because it should be exactly one-sixth the length of the string. If you refer back to our previous discussion of partials, you will realize that sympathetic vibration will cause this to emphasize the sixth partial of the open string and any other notes with that same partial (or fundamental). For the player, this suggests that the attachment of fine-tuners that shorten the length of the string behind the bridge is detrimental to the sound. (Make sure any fine-tuner is the type that does not shorten the string length behind the bridge). This fact also suggests that changing the tailpiece can affect the string length and thus should only be done at the suggestion of a professional luthier or restorer. Tailpieces aid in damping excessive vibration that would cause extra and undesirable frequency components in the sound, called “wolf” pitches. But choosing a tailpiece of a different weight can change the sound drastically (Beament, 38). In any case, now that we have completed the introduction, we will proceed to the portion of the violin that the player controls.

## **Tone: That Which Possesses the Capacity of Change**

It is often said that tone is the first presentation of a good violinist, and so, not surprisingly, a myriad of schools of thought exist on tone production and bow technique. What makes each of these schools of thought valid and how do they contribute to a full palette of colors? These are the questions I will attempt to answer in this section on Tone.

We have already discussed the basic motions of a plucked string. The motions of a bowed string also generally follow the Helmholtz pattern, but with a specific difference: the motion of the bow sustains the vibration pattern. The bow and the string operate in something that is called a slip-stick mechanism (Cremer, 12). At the point where the bow touches the string, the string has exactly two phases: sticking and sliding (Cremer, 55). When the bow catches the string, it begins to pull the string sideways at the same speed as the bow. This is referred to as the “sticking friction.” At a certain point, the force of elasticity in the stretched string becomes too great for the bow to maintain its frictional hold on the string, and the string “snaps back” (Johnston, 121) to the far side of its vibration pattern (due to its tension, of course). The snap is called “slipping” or “sliding” friction. The resulting motion and sound simulates a sawtooth wave. (See Diagram 9).

The full process (sticking and sliding) happens at a rate directly proportional to the frequency of the string. (Ex. 440 cycles per second when playing the open A string.) Of course slipping friction is always far less than sticking friction. The time interval of sticking friction is longer than sliding friction because the sticking friction

is fixed by the speed of the bow, and the sliding friction is fixed by the pitch of the note (Cremer, 122)(Beament, 16). In other words, the entire period must take  $1/440$  of a second on the open A string. If the string moves more slowly during the sticking phase, it must move more quickly during the slipping phase to make up the difference in time. The most obvious application of this concept is that if the weight on the bow causes its frictional force to exceed the maximum sticking friction, the sliding phase will be impossible due to lack of time and a grinding sound will result (Cremer, 10)(Johnston, 120). In any case, the sawtooth wave can look very different based on the varying times of sliding friction. Here is one possible diagram of the relative time of the two phases:

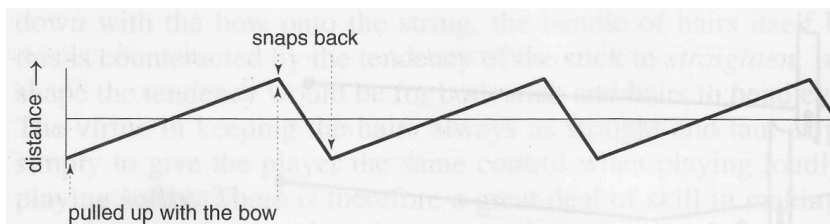


Diagram 9 (Johnston, 122).

The quicker the sliding friction, the sharper will be the triangular shape of the waveform (and the greater number of upper partials). We will count this type of motion primarily as forced oscillation since the time allowed for sliding friction is virtually non-existent and the player puts in constant energy (bottom example in Diagram 10)(Cremer, 82-83). The faster the bow speed and lighter the weight (as long as minimum weight is achieved), the longer the sliding phase becomes, and the closer the wave approaches to a perfect sine curve and nearly free oscillation in

which the string pursues its own oscillation independently of the bow (top example in Diagram 10)(Cremer, 11, 15, 156). If the bow were placed at the middle of the string, the time for the displacement of the string would indeed be equal in both directions and a nearly perfect sine curve would result (Cremer, 39). This accounts for the very mellow sound of bowing *sul tasto*. Here are some examples of different waveforms resulting from different times of sliding friction:

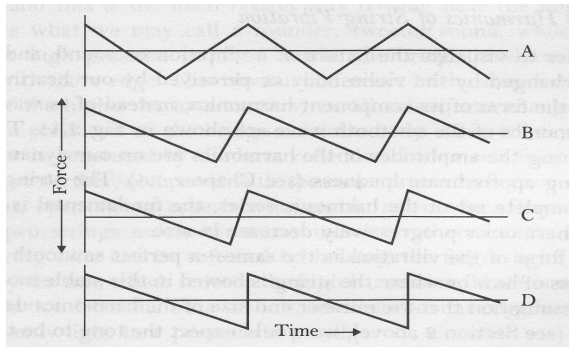


Diagram 10 (Beament, 19).

The problem with separating the two phases (sticking and sliding) is that they happen so quickly that the string is never a completely free of the bow nor completely controlled by it. Once the string is vibrating, the pulses and waves begin to stabilize,(Cremer, 175)(very much like the Tacoma Narrows Bridge) and the string can be viewed as being both in forced and free oscillation. The violin relies on neither of these types exclusively, but rather changes time proportions of each phase based on the player's versatility. Frankly, this is where the different schools of thought on tone production present themselves. We will discuss the two most extreme schools of thought; all the rest fall in between.

Our discussion of violin technique thus centers on the concept of what is considered “good” sound by different schools of thought and the physics that create it. Since we are primarily concerned with the machine that is the violin, we will begin with the physical motions that activate the violin, proceed to their direct effect on the vibrations, then to the affect of the vibrations on the perceived sound. Finally, we will examine the benefits and detriments to the player himself. The differences in schools of thought rely primarily on whether the violinist in question considers the string a forced oscillating object (American, Russian, and New York school), or a free oscillating object (Western European and Cleveland school).

The first school of thought propagates the view of the string as forced oscillation (although of course, teachers do not use this term). Forced oscillation involves the constant contact of the bow motivating the string into vibration. This school of thought places a premium on projection, meaning that good sound is considered to be strong, unbroken tone with somewhat of a thick texture and a full range of upper partials made by bowing close to the bridge. Achieving this bravura sound requires a significant amount of weight from the bow arm and a bowing location that is extremely close to the bridge. (Of course, these two requirements usually indicate a somewhat slower bow speed as well.) The tone is full and powerful, often with some extra noise due to great friction between the bow and the string. But the primary reactor to the extra force of the bow is the bridge. Here is why:

The weight given to the bow by the arm directly controls the amount of friction between the bow and the string (Beament, 22). As the frictional forces

increase with extra bow weight, they exert additional force on the bridge. The extra force results in secondary torsional waves. (Torsional waves are vibrations that twist the string.) As torsional waves hit the bridge and reflect back to the string, they often turn into regular horizontal vibrations and vice versa, increasing vibrating instability and the need for constant energy input by the player.

Secondary waves also affect the point of bowing (Cremer, 135). In fact, perfectly periodic motion only occurs farther away than  $1/15^{\text{th}}$  of the string from the bridge and closer to the bridge than  $1/7^{\text{th}}$  of the length of the string (Cremer, 53). Closer to the bridge, where there is more bow weight (Cremer, 50), the string moves in a highly irregular pattern (Cremer, 50) Why? The high pressure on the bow in combination with the up and down motion of the bridge generates secondary waves in a vertical direction. While these waves have very little effect on the sound, they do increase the force at the bridge and thus the difficulty of playing (Cremer, 164). They also create extra impulses traveling through the string past the bow and this causes the reduction of periodic motion extremely close to the bridge. Tiny, non-periodic vibrations disrupt the sound, increasing the roughness of the sound which is (in very small amounts) the signature of real violin playing (as opposed to electronically synthesized playing) (Cremer, 198). Since secondary waves and other additional waves are based on bending stiffness, these waves play only a small role in strings that have less stiffness. However, on the stiffer strings (or in high positions where the shortening of the string increases stiffness), bowing extremely close to the bridge is a very sensitive task (Cremer, 147). The extra torsion on the string at these frequencies can actually cause the same flattening

effect as excess pressure, and thus, playing must be treated with care (Cremer, 119).

Extra torsion also has an effect on tone color since any variation of the sinusoid wave will create variation in sound (Cremer, 135). According to Beament, "It is a riddle that to be perfect to our hearing, an instrument must be imperfect and behave irregularly." (Beament, 214).

In the forced oscillation model, the bow pulls the string far to the side and creates a sharp angle with the bridge. The sharp angle pulls the bridge sideways and increases the force on one of the bridge feet (and by virtue of the sideways vibration, both feet) (Beament, 19-20). The increased force from this sharp angle and the previously mentioned secondary waves induces the bridge into increased (and rather unstable) vibrational amplitude (Beament, 20). Vibrations of greater amplitude hit the body and transfer to the air, causing an increase in volume. The 2<sup>nd</sup> harmonic becomes very prominent (due to the previously discussed bridge foot motion). The higher harmonics also maintain a significant place in the tone (as shown by the sharp corner of the wave).

The violent motions of the bridge in this type of tone production contribute further to the non-periodic motion in the string. Since the production of overtones and torsional waves takes up a significant percentage of the sound, the bridge's vibrational axis constantly shifts back and forth in response to the high amplitude of the extra frequencies present in the sound. This causes the sticking and sliding motion of the string to become increasingly non-periodic as the bridge changes its angle with the string. The resulting waveform shows the resulting irregularities or

“hitches” in the sound for small and large amounts of pressure (See Diagram 11)  
(Cremer, 83).

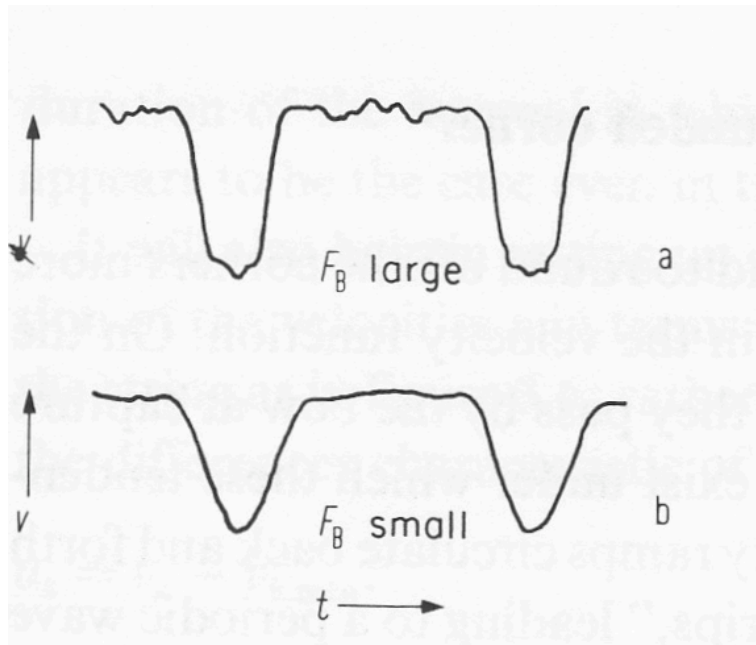


Diagram 11 (Cremer, 83). Diagrams for non-periodic variations in the sound with large and small amounts of pressure.

Instability of vibration causes the range between the minimum and maximum bowing pressure to become very small. This instability is caused by the difficulty of maintaining the sharp angle of the string with the bridge. The string constantly loses energy, and only through additional friction/bow weight can this angle be maintained (Beament, 17-18). However, this in turn produces extra tension, and producing high tension causes a complication. Since the disparity



between the amount of sticking and sliding friction increases with the bowing weight (Cremer, 72), the slipping phase becomes extremely time-sensitive. Since the sticking phase becomes longer as the player increases weight, the sticking phase can become so long that there is no time for the sliding phase and the entire period is forced to lengthen. When this happens, the pitch will flatten (Cremer, 106-107). Another problem happens when the bow itself becomes a node due to excess pressure (i.e. the string cannot vibrate at that location). When this happens, the soft bow hair reflects the wave on the string and creates unstable pitch (Cremer, 110). Sometimes the tone becomes scratchy when more than one capture and release occurs during each period (Cremer, 67). All of these effects should be avoided unless specifically called for in extended technique.

Therefore, when bowing close to the bridge, in order to both excite and maintain oscillation, the task of putting in the correct amount of weight becomes extremely sensitive. Even in expert bowing, when the bow is close to the bridge and pressure is great, the forces on the string at the bridge are such that the string cannot achieve a significant interval of free oscillation, and the player must continually put in a great deal of energy. We have already mentioned that energy is neither created nor destroyed. However, if the violinist must continually insert a great deal of energy, we must conclude that the energy is escaping from the system. How is it escaping?

Since the bridge is constantly pulling a vast amount of energy out of the string to facilitate its own vibration, some of this energy is lost to the bridge as heat with the constant changing of the axis of the bridge and torsional or secondary

waves. These internal waves cause a quicker decay of oscillation due to multiple wave reflections and interferences with the motion of the string. Finally, a large portion of the energy given to the bridge transfers into the body of the violin and thus to the air in the form of high-amplitude sound waves (Beament 20). This happens because, with greater pressure, there is greater friction and the string is displaced further to the side, increasing the sideways swing of the string and thus the amplitude of the vibration (Beament 18).

In accordance with these physical facts, the primary principle taught by the adherents of this school of thought remains that sound must be maintained by the constant energy of the player. In simple terms, nothing in the machine is stable. Significant losses of energy cause the player to do a great deal of work to achieve steady sound and vibration. Many teachers suggest doing this by feeling consistent friction between bow and string. (Beament 21). Indeed, a violinist who uses this method of bowing correctly achieves an unbelievable volume of sound and projection. The only real problem—besides extra work for the player—is that using the string as a forced oscillator only allows for a limited range of types of sound because it only utilizes the extreme end of the spectrum of string excitation. This is exactly what one hears in a violinist who adheres exclusively to this school of thought: one or two types of sound and a great deal of energy needed to create them.<sup>3</sup>

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<sup>3</sup> Most teachers refer to “bow pressure.” Every professional violinist knows what this term really means. It is not the idea to which I object, but the term. And for this reason: tension. It was Ivan Galamian who talked about the three parts of bow technique most clearly. He called them speed, pressure, and sounding point (Galamian, 55). I believe we would do well to refer to this as bow weight instead of

Now let us take a look at the free (or nearly free) oscillation model of playing. This school of thought views bowing speed (instead of bow weight) as directly responsible for increasing amplitude. The first and most obvious result shows itself in less bow weight and therefore, a greater percentage of time for sliding friction than the forced oscillation model (Cremer, 12).

The resulting ringing sound possesses a certain flexible singing character, fewer partials, and the capacity of great subtleties. It does, however, lack the highest level of projection. Bowing pressure is kept to a minimum and additional bowing pressure above minimum is added only when producing certain full tone colors. Otherwise, minimum pressure is the rule. The logic behind this theory is that, since the string is vibrating side-to-side, additional pressure above the minimum would dampen some of the amplitude.

The free oscillation model makes use of the efficient vibrating system of the violin strings and the relatively inefficient transfer of this vibration to the air. In order to start the string vibrating, the player must add extra energy. (This will be discussed in the section on transients). Once the string is resonating, energy input and output are relatively small when compared with the oscillating energy of the string (Cremer, 56). Since the bridge, body, and air remove only a small portion of the energy from the string, the violinist should not have to work very hard to keep the energy going. Only about  $1/10^{\text{th}}$  of the energy is lost during the sliding phase

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bow pressure because, aesthetically, the term "bow weight" suggests the use of larger muscles in the inner part of the back and shoulder in addition to the natural weight of the arm, while the idea of "pressure" suggests the uses of the small muscles of the hand and index finger.

when the pressure is low. The string loses  $4/10^{\text{th}}$  of its energy when the pressure is high, most of which transfers into sound (with some extraneous noise) and heat (Cremer, 60).

Let us look at the physics behind the free oscillation model. Raman supports the idea of minimum bowing pressure and demonstrates that a minimum bowing pressure or weight must exist for sound to be produced (Cremer, 72), given that friction must be created in order to pull the string sideways. However, even Cremer admits that no “qualitative change in behavior of [the] string is dependent on pressure above the minimum, even according to the forced oscillation model” (Cremer, 77). Pressure on the bow is not so critical except very close to the bridge. Everywhere else, you can work with different amounts of pressure without extreme consequences. Couple this with Lazarus’s research that for a significantly smaller time of sliding friction and thus a sharply triangular wave form, the bow weight must be increased exponentially (Cremer, 55). Now we see that research seems to support that additional pressure does very little for the sound and does quite a lot to overwork the violinist. My personal opinion on this point is that mere bow weight is expected to increase or decrease the sound level on the violin far too frequently. The most effective methods of varying dynamic levels and changing the sound or character fall to other tools possessed by the violinist, namely location and bow speed. This is in accordance with the free oscillation model. However, since high amounts of bowing pressure not only increase amplitude, but also increase torsional waves, we can conclude that in certain situations requiring projection or particular colors the forced oscillation model can and should be used. Thus, each school of

thought represents not an exclusive method, but rather a different side of the coin. The judicious use of both types of tone production and the wide range of combinations in between gives to the violinist a wonderfully large palette of unique colors and subtle phrasings. As one great teacher said “if all sound is ‘good’ sound, there will be only one color” (William Preucil). Musical expression and communication with an audience require the use of every tool at our discretion.

So then, the American school of thought with its more projecting, but somewhat rigid definition of “good” sound, and the more colorful, but softer and highly variable European sound both have their place. By way of the application however, it seems we can conclude that, based on the physics, the violinist should always determine the minimum pressure for the *desired sound* and use primarily a constant speed to maintain amplitude. Once the string is vibrating, the bow only has to replace the small amount of energy lost to frictional processes (Beament, 23). This encompasses both the American and European schools of thought and it enables the easiest method of producing both sounds. Since the arm and bow continue moving due to inertia, almost no energy must be added to maintain amplitude during the bow stroke. Although pressure will increase a bit when bowing close to the bridge, frankly, the concept of ongoing energy makes playing the violin seem much easier, and incorporates both schools of thought. Playing in this fashion creates just as much sound and much less exhaustion, a topic that will be handled in the following section about the physics of the body.

An interesting note regarding vibrato and sound can be added at this moment. When the player bows the string in such a fashion as to stabilize its

vibrations and create a ringing sound, partials comprise a significant portion of the sound. Since human hearing is more sensitive to large numbers of high frequencies and pitch detection is more accurate with increased number of high partials, only a narrow movement in the hand will achieve a vibrato that presents itself to the ear as being of considerable width (Philip Setzer). The fewer partials in the sound, the wider the vibrato will have to be to achieve the same result (Beament, 129). Since all partials fluctuate with the vibrato, the vibrato should move across both sides of the pitch, contrary to popular belief that one should only vibrate below and up to the pitch (Beament, 129).

Achieving this magical and effortless sound, according to Beament, involves creating “conditions in which the string stabilizes itself.” (Beament, 16). He also says “What we might colloquially describe as the quality of sound seems to be associated with minimizing amplitude variation during the sustained part of the note.” (Beament, 31). In other words, the amplitude of expert bowing changes very little except where a crescendo or decrescendo is desired (Beament, 130). This makes sense with the laws of energy because of a simple phenomenon. If an object which is already moving (think of a child on a swing) is periodically disturbed with the same amount of energy required to start the object in motion, the vibration will only increase in amplitude (Johnston, 224). The same is true for the violin string. If however, the violinist starts the string in motion and afterwards maintains bow speed throughout the stroke without an increase of pressure, the amplitude will remain relatively constant, since only a minimum amount of energy is being supplied by the bow to sustain the note. Unless the location of the bow is changed,

weight is largely irrelevant (Beament, 17). In light of this, it is wise to view each bow stroke as one event. The initial impulse excites the string into vibration and inertia, resonance, and the continued (but of necessity less than the first impulse) inertial dynamic of the moving bow and arm keep it in constant vibration. Only in the case of a volume or color change (which is of course very common) in the middle of a bow stroke must there be additional “messing around.”

In conclusion, in order for a violinist to possess full control of artistic choices and a complete tonal palette, s/he must be sensitive to the way s/he produces sound. S/he must be able to change not only the three basic parts of bowing, fluently, but also to change the type of sound within each one. So where do we go from here? I find it helpful to identify the school in which one has been trained. From this point, one should experiment in the opposite direction. Most players are surprised by their discoveries. What we should call “good” tone production actually consists of many sounds spanning the gamut of violinistic possibilities. Each school by itself, although dogmatically defended by its adherents, is lacking something of the full spectrum of sound. In terms of physics, I think it is self-explanatory that once a desired sound has been achieved, the least work necessary to maintain that sound should be pursued. Excessive effort causes constant instability and strain, and generally sabotages good technique. In one author’s words, “The most complete solution [of tone] views the dynamic process of bowing as periodic, but superimposed on a static displacement generated by a *minimum* frictional force.” (Cremer, 72).

What must follow this discussion of the basic physics of tone is an acknowledgment that different violins do affect the mechanics of tone production. The responsiveness of a given instrument is not only evident in the ease of the initial speaking of notes (Beament, 94). The energy needed to maintain amplitude during a note, also varies with the efficiency of the instrument. There is a large difference in efficiency between most French and Italian instruments. The difference in ease of playing lies in what percentage of sound energy the body takes from the string. If the body “takes” more energy out, the player will have to work harder (Vuillame). If it takes less energy (Amati), the player will not have to work so hard. If the instrument is efficient, you will only have to add a little energy once the initial vibration has begun. If it is not, you will have to add a bit more. Efficiency does not make an instrument good or bad, it merely changes the amount of input required from the player. Some of the finest projecting instruments require high levels of input and some require low levels. What does make the instrument good or bad involves where the energy is going. If it escapes merely as heat, one has a poor instrument. However, if it escapes into high amplitude sound, one has found an excellent instrument that will be capable of projecting. The truth is that all good instruments, however, are relatively efficient when one compares them to the fairly awful instruments that most children and students play.

It really is up to the player to determine what kind of efficiency they prefer, because only a very small portion of projection is based on volume (amplitude). Most projection relies instead on a strong fundamental with the combination of certain important lower partials (Beament, 74). In general, instruments with strong



resonances in the 1-4 kHz range will project well, because this is the range in which the human ear is most sensitive. What is perhaps most interesting about partials and projection is that the very highest partials are not necessarily very useful in increasing projection because the wavelengths are so short that they are quickly absorbed by room boundaries (Peterlongo, 95), and the highest partials have shorter decay times (Cremer, 26). Rather the partials that matter most are those up to the 6<sup>th</sup>. The finest instruments remain strong in amplitude up to this partial and then gradually decay up to the 20<sup>th</sup>. Each violin will have certain “dead” notes, or notes that the player has to work harder to create (Beament, 77). The really good player can produce excellent sound from a “cigar box,” but, as we all know, a truly excellent instrument does inspire confidence and enables the violinist to reach his potential, especially at the limits of technique (Beament, 94).

### **Color**

Color represents one of the greatest creative challenges of violin playing. It is largely determined by the presence of different partials in a given note (Peterlongo, 37), and while the violin is capable of many colors and timbres, the easiest way to explain the phenomenon of timbre is to compare the spectrum of other instruments to the violin’s spectrum. Diagram 12 shows the spectrums for the flute, oboe, and violin (Johnston, 92-93, 99).

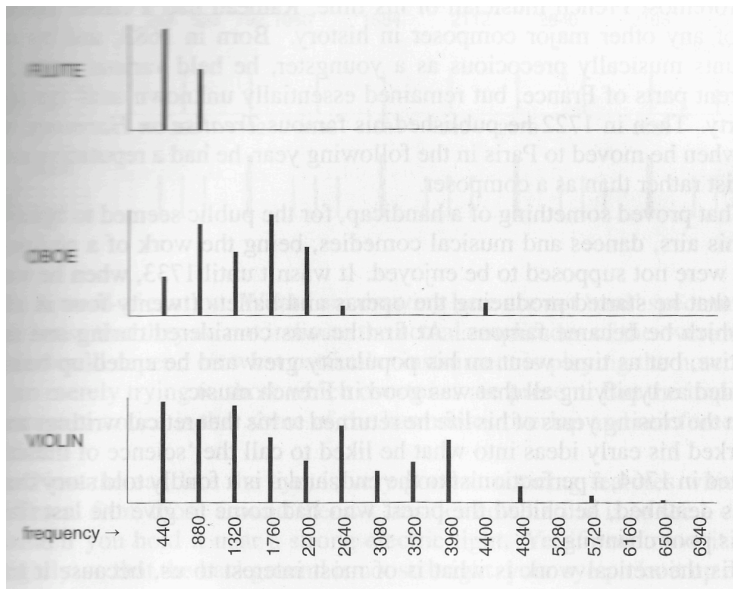


Diagram 12 (Johnston, 99). Spectra of three instruments: in a standard spectrum, the amount of energy present at each frequency (partial) is indicated by the length of the line, with the fundamental being the lowest (furthest left) frequency.

Certain instruments, like the flute are missing specific partials, and this creates their easily recognizable timbre. When examining the spectrum of the flute, one can see that the second partial is very strong, but few subsequent partials have a significant presence. This creates a fairly simple wave modification “bump” to the simple sine curve that one would get with the fundamental alone. The oboe lacks or is weak on nearly all of the even partials. This accounts for its nasal timbre (Johnston, 199). The violin shown is relatively strong until the 9<sup>th</sup> partial of A-440. As previously mentioned, the very best old Italian violins are strong at least until the 6<sup>th</sup> harmonic (See Diagram 13)(Peterlongo, 100-101).

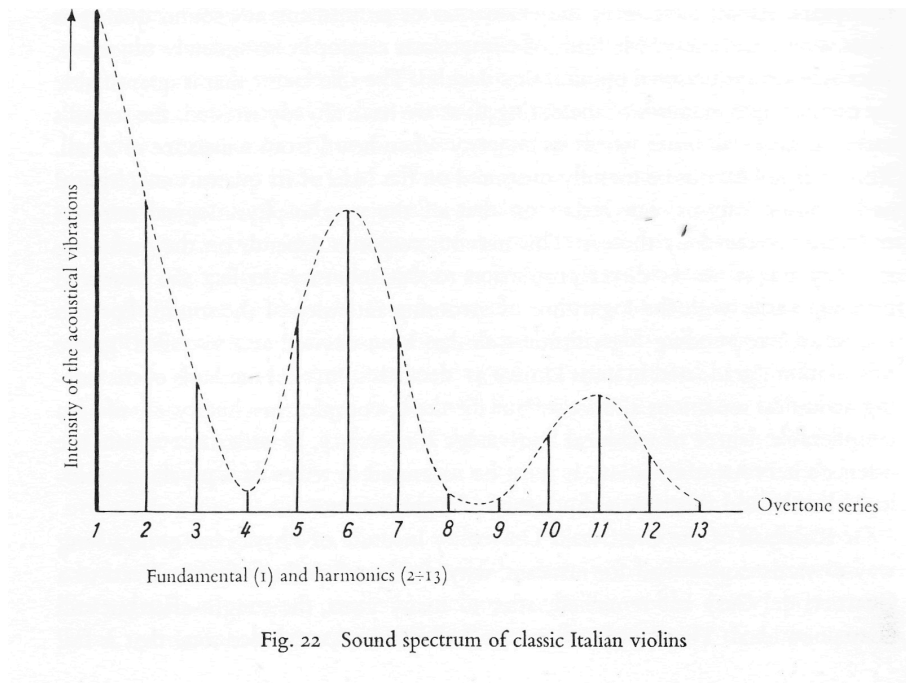


Diagram 13 (Peterlongo, 100). A different view of a spectrum: the fundamental and the harmonics are all “partials.”

Many modern violins instruments are designed to strengthen partials up to the 20<sup>th</sup> in the hope of projection, (see Diagram 14) but as we have already seen, this has a tendency to backfire because the shorter wavelength of the highest partials is absorbed by the walls of room and the correspondingly lower amplitude of the fundamental is unable to compensate.<sup>4</sup>

<sup>4</sup>. [Footnote: Remember, only a certain amount of energy is in each system—in this case, the violin. If more energy is used for the upper partials, there is less for the fundamental.]

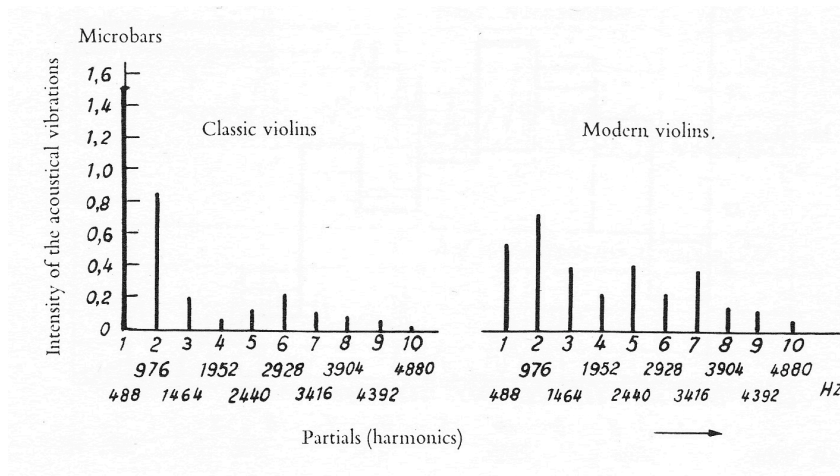


Diagram 14 (Peterlongo, 101). Two more spectra; microbars are units used to measure pressure, in this case, sound pressure.

In addition, a note with large numbers of upper partials (higher than the 6<sup>th</sup> harmonic) (Peterlongo, 37) will sound “brassy,” and this is usually not desirable. As Johnston says, “A bit of [overtone] dissonance is like spice,” but too much can distract from the fundamental (Johnston, 241). Since the violin is least efficient at coupling the lower pitches to the air, the best violin makers have tried to emphasize the fundamental without suppressing important partials (PL 380).

In addition to this, we find that steel and gut strings have vastly different colors because their elastic potential energy is different. The steel string will yield a greater number of upper partials because of its increased tension and significantly decreased internal damping, while the gut will turn its lesser potential energy into an emphasis of the fundamental due to its lower tension and heavy internal damping. Of course, too much emphasis on the fundamental with few partials will

lead to lack of projection, so there must be a happy medium which depends both on the player and the instrument (Johnston, 83).

We know that different instruments produce different timbres, but how does one produce different colors on the same instrument? There are many factors that influence color. We will discuss them individually for the sake of simplicity, recognizing that the combination of multiple factors yields a vast array of colors. The primary factors include bowing or plucking point, modes of vibration, bowing speed, string elasticity, fingerings, finger placement, and vibrato.

The first factor involves bow placement. Bow placement matters for two reasons. The first reason is our old friend inertia. Since the corners created by the bow are rapidly moving up and down the length of the string and gradually rounding out as they do so, the corners are sharpest where the bow touches the string. If the bow is far from the bridge, the corners have a chance to round out before reaching the bridge and thus the sound that is transferred to the bridge has fewer high partials. If the bow is close to the bridge, the corner is steep when it hits the bridge and transfers the vibrations. In this case, the sound often contains partials up to the 20<sup>th</sup>! (Johnston, 124).<sup>5</sup>

The second factor stems from different modes of vibration in a stretched string. The bridge and the nut both form a node for the vibration of the string. This means that in theory, at the point of the bridge and the nut, there is no movement, and this is important in establishing the wavelength of the fundamental. If one is

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<sup>5</sup> It should be noted that corners are never perfectly sharp because the bow is not only one point, but a band, but this prevents the tone from being too strident (Cremer, 26).

producing an “A” with a certain length of string, it is not possible for the node at the ends of the string to move. If it did, a different note would be produced. Let us apply the logic of this to partials. The place where the string is plucked or bowed represents an antinode because the string must travel a certain distance back and forth at this point. Since the place where the string is plucked or bowed automatically becomes an antinode, if the string is plucked at a length equal to the *node* of a partial, that partial simply cannot vibrate. For example, if a string is plucked at  $\frac{1}{4}$  of its length, the 4<sup>th</sup> partial will not sound, etc. (see Diagram 15).

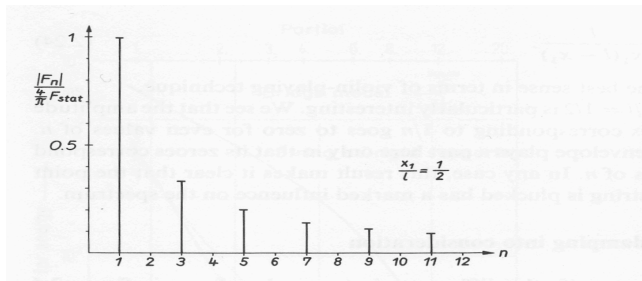


Diagram 15a, (Cremer, 25).

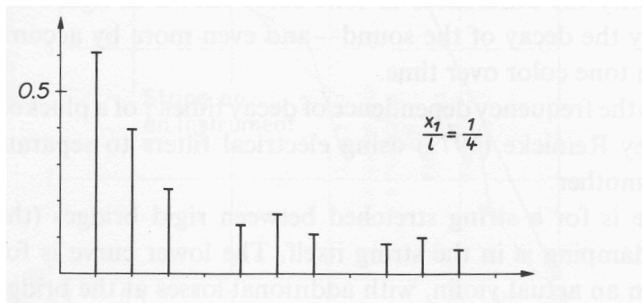


Diagram 15b, (Cremer, 25).

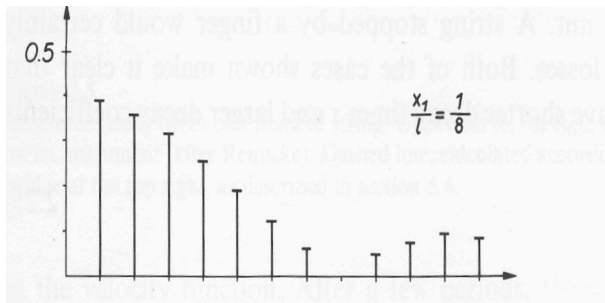


Diagram 15c, (Cremer 25).

*Diagram 15a: Partial's amplitude when bowing at the node of  $F \times 2$  (second partial).*

*Diagram 15b: Bowing at the node of  $F \times 4$  (fourth partial).*

*Diagram 15c: Bowing at the node of  $F \times 8$  (eighth partial).*

The same principle is true for the bow. If the bow touches the string at the node of any given partial, that partial will disappear from the sound. (Cremer, 40)

Let us say that we are bowing at  $\frac{1}{4}$  the length of the string. Not only will the fourth partial disappear, but every multiple (#4, #8, #12, etc.) of that partial will also disappear because the nodes of these partials are in the same place (Cremer, 25).

Bowing at the node of a partial also tends to emphasize the next partial in the series because it has an antinode at the place of bowing (Cremer, 50). Indeed, bow placement was one way that the great masters created color. Now, of course, bowing at  $\frac{1}{4}$  the length of the string is rare in first position, but it is quite common in high positions. One can see why playing *flautando* over the fingerboard (fairly close to the 3<sup>rd</sup> harmonic, and thus emphasizing only the second and fourth) creates a spectrum that compares to that of the flute. The most common colors in bowing actually happen between  $\frac{1}{8}$ <sup>th</sup> and  $\frac{1}{16}$ <sup>th</sup> of the way along the string. This means that, when bowing at  $\frac{1}{8}$ <sup>th</sup> of the string, the 8<sup>th</sup> partial and its multiples will be

missing, but since partials higher than the 7<sup>th</sup> or 8<sup>th</sup> are not as important, their absence is somewhat negligible (Cremer, 52). There is always room for expansion into different colors by choosing a different point of bowing; the finest players make frequent use of this technique.

Bow speed also affects tone color because it controls the distance the string moves. We have already pointed out that the speed of the bow controls the speed of the string during the sticking portion of the stick/slip cycle. Let us follow the logic further. Sliding friction increases or decreases based on bow speed (LC 59). If the bow speed is very fast, the string will reach its point of greatest friction more quickly and the sliding phase will begin. The sound will generally be more round because longer sliding phase creates a somewhat rounded triangular wave. If the bow speed is slow, it will take longer for the string to reach its maximum point of friction and the sliding phase will have to be very quick to make up the deficit in time. Thus, the wave will be more triangular (more energy to higher partials, and a brighter sound).

The elasticity of the string is the fourth factor that affects color. Each string has a different elasticity and thus produces a different harmonic content. The stiffer the string, the more ability it has to vibrate strongly at the high frequency partials. Less elasticity creates higher frequency partials (because of less internal damping). This is why the high E string sounds so shrill (Peterlongo, 39). Greater elasticity allows for fewer high frequency partials, but an excellently strong fundamental and lower partials. This means that string choice can matter quite a lot. Within each



string, elasticity changes according to how short the string is made by the finger.

The G string, for example, has so little elasticity in the highest positions that the fundamental becomes almost inaudible due to the relative strength of the partials, making the notes virtually unusable. This means that not only does string choices matter, but position choice (how high on the string one fingers) also matters a great deal. This elasticity principal extends to the higher positions on the other strings as well. More tension on the string equals less elasticity.<sup>6</sup>

This leads us to the fifth factor influencing timbre, that of fingering. Every excellent violinist knows that fingerings are crucial in the control of color. This is largely for both the reason of elasticity previously mentioned, but also for the additional reasons of reflecting point and vibrating mass. Use of fingering in coloring the sound can be demonstrated by playing the same note on different strings. Let us take the example of E5. The open E string will contain a large number of upper partials and a strong fundamental because it has the most elasticity (due to its thinness and length and the hardest reflecting point (the nut))(Peterlongo, 39). The same E played on the A string will have a less elasticity and greater tension, and thus more upper partials and a weaker fundamental. However, due to the softer reflecting point (the finger) and a smaller and shorter vibrating mass with more

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<sup>6</sup> As already stated, types of strings affect elasticity and overtones. In gut strings, almost all the energy put into the string is used up in deforming it. Internal damping of excessive upper partials causes it to do more quickly what the player wants (fewer high frequency overtones)(Beament, 212). With this advantage goes a greatly increased possibility of the flattening effect since bow pressure need only be half what it is for a steel string (Cremer, 119). Steel strings have less internal damping and contain large amounts of strident, high frequency overtones. Synthetic core strings are a good compromise for most players (Beament 215).

frequent reflections between the bridge and the finger, the highest partials will decay quickly (Peterlongo, 85). These frequent reflections also lead to less volume of both the partials and fundamental, and to a shorter decay time since each time the wave is reflected, it is reflected back with less strength (Cremer, 107). The E5 on the D string would normally contain more high partials due to differences in stiffness and tension, but again, the vibrating mass and reflecting point change the sound. Since the vibrating mass is now very small (as the distance between the finger and the bridge is short), frequent wave reflections require shorter string length but will also include the possibility of the bow damping out the 4<sup>th</sup>, 5<sup>th</sup>, or 6<sup>th</sup> partial and its multiples by bowing at a node. The sound will therefore be soft and a little hollow with a somewhat weaker fundamental. When playing the same E on the G string, the upper partials become so prominent as to make the fundamental unstable, and one of these low and important partials will almost certainly be missing due to bow placement. Coupled with the huge amount of tension on the string and the very small vibrating mass, the color changes drastically and the note is only rarely used.

Since each string has its own mass and elasticity, one of the secrets of the greatest violinists has been to manipulate fingering choices so as to place each phrase on a single string and maintain nearly the same color throughout. As you can see from the previous discussion, this makes sense. It usually means that the vibrating mass changes only gradually (rather than the abrupt change of mass due to changing strings), and that the tension and elasticity contain fewer variations. Thus the harmonic content also contains fewer variations and the tone color becomes more cohesive. Where this is not possible, fingering in similar positions on

different strings forms an acceptable solution since at least adjacent string have similar elasticity in the same positions. The minimum variation possible is usually best, since it approximates the human voice in creating a comparative quality of line throughout a phrase.

The sixth factor involves finger placement. The fingers themselves create a reflecting point for the waves traveling along the string, and thus, the finger the player uses can create a more or less rounded corner in the reflected wave. Using the fleshy part of the finger rounds the corner and creates a warmer sound (damping out the highest partials), while using the more bony tip often keeps the corner sharp (allowing the presence of upper partials by creating a stiffer reflection point that does not absorb the shorter waves). Of course, even different fingers on the same hand have harder and softer characteristics. Using the pinky instead of the ring finger can even create a different color. Different types of finger placement and finger choices increase breadth of expression.

One final topic regarding timbre must be addressed: that of the voice and the violin. From the oldest musical treatises, we know that the violin was first seen as an imitation of the human voice. This makes sense from the standpoint of physics. As Ian Johnston says, “[The vocal tract] acts in some ways like the box of a violin, in which the enclosed air vibrates in sympathy with the strings. The strings’ vibrations have a complex harmonic structure; but the box has its own set of resonances and that determines which of the frequencies—fundamentals or overtones—will couple efficiently to the outside air.” (Johnston, 329).

The studies of Helmholtz, Paget, and Willis prove that the vocal tract can actually change its own resonances to produce different vowel sounds. Of course the violin cannot change its resonances, but makers have made compensations. Let us look at the phenomenon of vowel sounds. Robert Willis discovered that every vowel sound is connected with a particular frequency related to resonance in the oral cavity (Johnston, 325). Helmholtz noticed that the first 6-8 harmonics came out in various frequencies based on what vowel was being sung (Helmholtz, 104). He also realized that this was primarily due to a resonance caused by the shape of the vocal cavity. When the tongue and the soft palate changed shapes to create different vowels, the resonant frequency of this cavity also changed (106-7 Helmholtz). Helmholtz's discoveries led him to believe that each vowel had one important vocal tract resonance, but Paget managed to prove that 2 resonances existed for every vowel (Johnston, 325). Diagram 16 shows the two resonances—which in vocal acoustics are called “formants”—for every vowel.

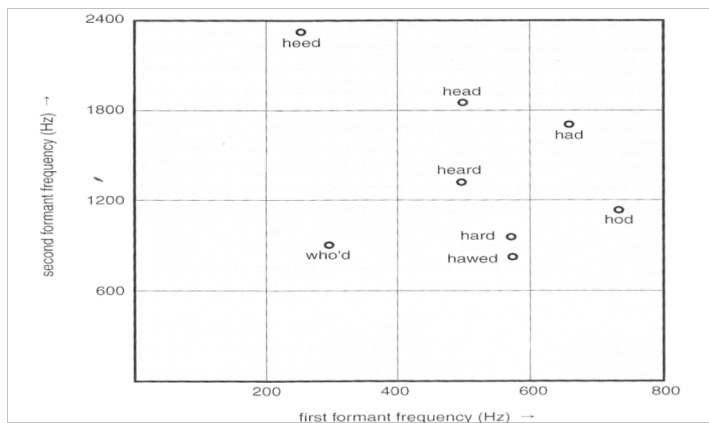


Diagram 16 (Johnston, 331).

While the violin will never be able to produce vowels as clearly as the voice, from this we learn that strengthening different partials actually changes the vowel sound (Johnston, 330-331). I believe in a certain degree it is possible to mimic this on a violin. We also see that at a certain frequency range, the fundamental pitch is above the first resonance required for the vowel sound to be clear. This is why it is difficult to distinguish the vowel sounds of high sopranos, and also to produce more than one vowel-like sound on the violin's high E string (Johnston, 335). Sometimes singers also distort their vowel sounds to put energy into an overtone range in which other instruments are not strong so that the singers can increase projection. We also see this happening with violinists who must project over an entire orchestra. Fewer colors and vowels are possible where extreme projection is needed (Johnston, 239).

Three rather interesting items of color remain: harmonics, ponticello, and *con sordini*. Harmonics and artificial harmonics are created by causing the string to vibrate primarily in the mode of the 2<sup>nd</sup>, 3<sup>rd</sup>, or 4<sup>th</sup> partial, etc. On the violin, this mode of vibration is achieved by light placement of a finger on the node of the partial in question. The partial with a node at this point then becomes the strongest amplitude vibrating along the string. This throws the string into a different mode of vibration, which continues as long as the finger remains in place and (due to inertia), often for a moment afterward (Johnston, 328).

Ponticello is a rather funny color created when the pressure and speed necessary to produce the fundamental become impossible due to bow placement nearly on top of the bridge. The resulting instability of the upper partials leaves out

the fundamental, but oddly enough, in a puzzle that has not been conclusively solved by science, the ear and brain supply the missing fundamental as the difference in frequency between the  $f_2$  and  $f_3$  (Johnston, 125).

Finally, the introduction of a mute (*sordino*) on the bridge also changes the color because the natural frequency of the bridge is lowered by having a greater mass and the higher harmonics are subject to increased damping. Reinke says that a mass of 1.5 grams pulls the bridge resonance down to only 1700 Hz. A mute is 4 grams so this makes the resonance much lower than this, and weakens the partials above the resonance point. The result is the production only of the 1<sup>st</sup> up to the 3<sup>rd</sup> or 4<sup>th</sup> partials (Cremer, 241), which accounts for the dulled sound.

Many violin pedagogues include vibrato as a way of changing “color.” Vibrato does indeed change the type of expression, and should be used as another tool in the service of musical communication. However, vibrato does not change the harmonic spectrum in the strictest sense. Rather, the gentle undulation of frequency draws the ear away from any strident or shrill sounds, particularly on the high E string (Beament 129). The only change in the partials comes from finger placement. As previously discussed, a finger placed on its tip creates a harder wave-reflecting surface and a finger placed on its pad creates a softer surface for reflecting waves.

Vibrato does change the overall feeling of the music based on speed and width, and this—rather than a change in the harmonic spectrum—is what most instructors mean by “changing color.” Vibrato also “covers many sins.” The violin varies in its regularity of pitch by a 1/10 of a semitone during normal bowing without vibrato, so it will be obvious that the introduction of vibrato (usually

around  $1/6^{\text{th}}$  of a semitone) will draw the ear away from this anomaly and make the sound more acceptable (Beament, 124).

### **Transients and Articulation**

No discussion of the violin would be complete without a discussion of articulation. In general, I prefer to split types of articulation into two categories: left hand articulation and right hand articulation. This is not to say that they cannot be used together, but in general, one type for each note is sufficient. Articulations generate sounds that are referred to as *transients* in physics, because they are typically shorter than the continuing sound. Transients always precede periodic oscillation, and quickly settle down into a steady state of vibration. Different transients produce different types of articulation. Perhaps the most surprising fact about transients is that listeners identify various instruments and players primarily by their starting transients and not by their continuing sound (Cremer, 184).

The changing sound of a transient comes from the process of stabilizing the amplitudes of the string (Beament, 127). Since the corner must move up and down the string, it takes a bit of time for the ends of the string to spring into motion (Cremer, 185). Sometimes it takes several hundred periods before perfectly periodic oscillation begins (Cremer, 190). The duration of the transient relates to both the mechanical resistance of the string and the player's own manipulation of speed and bow weight (Cremer, 148-149). Most players speed up imperceptibly from the beginning of a note or change the pressure during the transient (Cremer,

189, 186).<sup>7</sup> Unfortunately, no equipment exists to accurately record and analyze the transients of famous violinists. Even today's physicists rely on mathematical equations and proofs to explain this phenomenon and the differences between instruments and players (Cremer, 191).

The string's behavior while trying to achieve periodic oscillation is far from stable, and creating instability before a note will cause the transient to be additionally unstable (Cremer, 185). This means that except where tonal or dynamic variation is desired, the best and simplest solution to technical passage-work is simply to articulate, then relax, articulate, relax (Soovin Kim). Let me explain the physics of this approach. I have mentioned that increasing speed or pressure throughout the stroke destabilizes the tone. Less than perfectly stable tone presents a problem for clarity of articulation, because when the amplitude of the string is unstable on one note, inertia causes the unstable vibrations to continue into the following note. Clear articulation becomes very difficult. As in speech, the clearest articulation comes after the decay of the previous word. Therefore in playing, we find that the clearest articulation comes after the slight relaxation of the previous note. Pushing through to the end of the note causes constant instability and creates a great deal of difficulty in creating a simple transient for the next note. Of course, in slow passages, this is not quite so crucial and phrasing and dynamic variations come

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<sup>7</sup> Interestingly, torsional secondary waves decrease the time of starting transients on the violin because even a torsional wave is string excitation and, much like an athlete rolling over after impact, these waves bring about the compliance of the bow hairs in longitudinal waves by transferring vertical vibration into horizontal vibration (Cremer, 190, 192). Of course for the player, this has little relevance to technique, but it is interesting to know.



into play within a single note. I am really referring to the most difficult and technical fast passages.

Starting transients (except in the case of extremely violent accents) contain the same partials as the pitch that follows, but in different proportions from the sustained note that follows (Beament, 127).<sup>8</sup> Usually the transient is so short that we do not notice that it does indeed have pitch (Beament, 128), but this pitch is actually just long enough that a fantastic violinist can detect it and fix intonation before the audience realizes that a note has been played out of tune. We know that this is possible since it takes 12-15 milliseconds to detect pitch at higher frequencies, and it can take as long as 1/20<sup>th</sup> of a second (50 milliseconds) to create an excellent starting transient. Lower (G string) pitches usually take 3½ vibrations for pitch to be detected, and therefore, may also take more time to fix than high ones, but a violinist should still be able to fix the pitch well within the transient time. (Beament, 128) (If the transient takes longer than 50 milliseconds, it usually just sounds like a crescendo: Beament, 24). The shortest time it takes a normally bowed string to reach normal vibration is 40 milliseconds and up to a ½ second (500 milliseconds) for a string bowed close to the fingerboard. Time can be decreased by adding bow weight, but the transient will also increase in volume (Beament, 128). Since string and bow impedance come into play, it is easier to start a note in piano at the tip of the bow where the hair is less flexible and thus less likely to absorb kinetic energy (Beament, 24).

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<sup>8</sup> In the case of violent accents, precession, which will be discussed later, actually causes all of the partials to lower in frequency.

## Articulations with the Bow

There are three types of articulation with the bow. The first is the difference between up and down bow, or separate bowings (Peterlongo, 79). When the bow changes from down to up, the direction of the string's slip-stick vibration reverses. If you refer back to our discussion of inertia and the tendency of objects in motion to stay in motion unless acted on by a given force, you will realize that this transition takes a split second to come about. This gives us the transient or articulation between bow strokes (Cremer, 43). Significant amounts of energy are only lost if the player adds more friction by too much pressure or speed at the end of the bow. Control and steadiness is of utmost importance (Beament, 25).

Bowing close to the bridge produces articulations of great clarity and strength. This is due to the very short distance the notch<sup>9</sup> in the string produced by the bow must travel before contributing to the vibrations of the bridge. Since the notch is right next to the bridge, inertia and elasticity have no time to dampen the vibrations before they reach the bridge. Nearly all of the upper frequencies of the string thus transfer directly into the sound. However, when bowing far away from the bridge, the distance the notch must travel from the bow to the bridge allows inertia and elasticity to round off the corner, making the angle of the string less extreme and eliminating the highest partials. Therefore, bowing away from the bridge produces a less pronounced articulation, (Johnston, 124). An interesting

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<sup>9</sup> Refer to discussion on pg. 11.

phenomenon occurs when a violinist changes bow without an audible transient. In this case, either the transient is so soft that the audience does not hear it or, in a louder passage, the string has achieved a moment of nearly free oscillation so that the switch in direction merely coincides with the string's own vibration and no audible transient exists. This is the secret of the invisible bow change.

The bow itself acts as a damper for the string due to friction, making the staccato and martelé of separate bows possible (Cremer, 56). Diagram 17 shows a waveform created by a single martelé stroke (Cremer, 190).

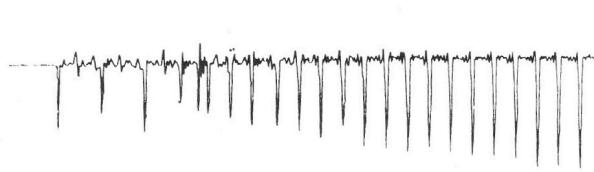


Diagram 17 (Cremer, 191).

Increased friction (due to increased bow weight and decreased bow speed) causes a dynamic increase near the end of the stroke (Cremer, 190). The bow stops between strokes making transients strong and crisp. At 230 Hz, it takes twelve periods to complete the transient of a martelé stroke (51 milliseconds, or just over  $1/20^{\text{th}}$  of a second). Decreased bow pressure after the transient is a requirement for martelé, as is the maintaining of a constant speed throughout the stroke (Cremer, 190). In actuality, the explosion of speed from the hand of the violinist brings this about naturally. Although the end of the stroke slows down, usually it is only the result of the frictional processes and due to no conscious thought of the violinist. In

this sense, the martelé stroke forms a perfect example of the articulating and releasing principle.

The next type of bow articulation comes from crossing strings. Since all strings take time for their vibrations to become periodic, it takes time when changing strings for the new string to achieve this vibration. The change can be gradual (when playing slow and legato) or abrupt, when using the wrist as a lever to change the bow angle abruptly and create a clean articulation during a fast arpeggio passage. Sometimes, in extremely fast arpeggios, where the string sometimes fails to respond, the left hand articulation on certain notes can encourage the string into motion.

Before we talk about the third type of bow articulation, we must talk about a particular phenomenon called precession. Precession is essentially circular motion of the string around its own axis. Ideally, the string vibrates back and forth in horizontal motions, but during precession, additional vertical and oblique vibration gets in the way of the simple back and forth vibration. Precession is directly responsible for the articulation in pizzicato, coming into play when a string is plucked and vibrates somewhat vertically as well as horizontally. The mass of the string determines how long the force continues to operate after the initial attack. A certain amount of precession is unavoidable in pizzicato, but of course it distracts from the regular, horizontal vibration of the string. The bridge vibrations become more vertical during pizzicato than during bowing because of this vertical movement, so the vibrations die out quickly (Cremer, 46). The string is further displaced than bowing, but the plucking finger is round, so there are no sharp

corners and, in combination with the long transient and quickly decaying harmonics, pizzicato from most violinists tends to sound a little dull. Vibrato has only the tendency to absorb extra string energy by damping secondary waves, so, except when it vibrates the note into tune (as is unfortunately often the case), vibrato does little to extend the length of the note, although it does beautify the sound.

The best method of ridding oneself of the unpleasant problems of excessive vertical vibration of the string during pizzicato is to pluck as parallel to the fingerboard as possible. This means that the finger must be deep beside the string, not underneath it, and not above it. Only then will the string and the bridge continue to vibrate similarly to bowed notes, horizontally (Beament, 30) The violinist must keep in mind that each string is at a different angle on the bridge and when plucking four note chords, the pizzicato must be deep on the finger and the arm must accomplish an arc similar to that of the bridge and the fingerboard. As with bowing, volume and harmonic content are determined by proximity to the bridge. In order to allow the plucking finger to clear the string cleanly and simply, many players find it useful to pluck a four-note chord at a 45-degree angle to the bridge.

In bowed notes, precession also occurs, but in expert playing it happens only minimally. Precession is generally undesirable because it sets up excessive secondary waves both along the length of the string and along the length of the bow hair. Thus, it is wise to start the bow from the string instead of from the air, since any vertical motion ("dropping" the bow onto the string) will create precession

(Beament, 30). Orchestra and chamber music players will also attest that this makes it easier to achieve similar transients, aiding in a convincing unison. Starting from the string is the only way to insure that two players will start from exactly the same height: it takes gravity out of the equation.

This brings us to the third type of articulation with the bow, which uses precession and is called ricochet. Ricochet requires that the bow must leave the string vertically after a bounce and return to it through gravity. At this point I must insert a small paragraph on the overall physics of the bow. The reason why this has not been inserted sooner is that except for tightening and loosening the bow, the player cannot change very much about its physics. However, in order to understand the bouncing of the bow, we must first explain how energy travels from one object to another. Let us start with an explanation from Ian Johnston's book with a simple and familiar object, Newton's cradle (Diagram 18).

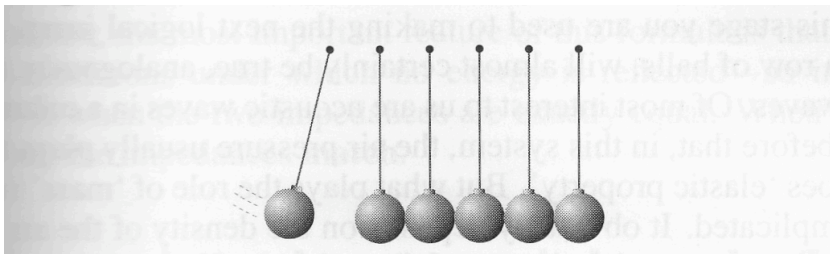


Diagram 18 (Johnston, 179)

If the steel pendulums are identical, energy travels through all of them with ease and back again. If they are not identical and one is larger or smaller than the other, different things happen.

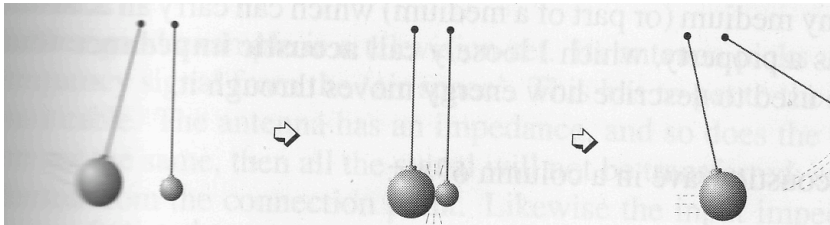


Diagram 19 (Johnston, 179)

In both cases, the moving ball keeps a portion of its energy because mechanical resistance (impedance) makes it impossible for all of it to transfer. But it is not this simple. What is actually happening in the row of steel balls is that, when the energy is transferred down the row, it happens because each ball is compressed ever so slightly. If a ball is heavier, it has more inertia and thus a slower compression. The small time delay causes the first ball to bounce back instead of stopping (rather like jumping on a bed that is too hard). The problem becomes even more complicated when one realizes that the elastic properties must also be the same as the mass if no energy is to be reflected. This, in a nutshell, is why the bow bounces after hitting the string. The mechanical resistances (both the mass and the elasticity) of the two objects are vastly different, and the energy that does not transfer into the string must be reflected back into the bow. Of course, a portion of

the energy is lost into the string vibration and into the air, so the energy that returns to the bow will be somewhat less than the original amount of energy. The player must supply the difference (Johnston, 179-180).

If the string precesses while playing, the bow hair will interact with the vertical vibration (because of its elasticity) and sound energy will be lost or dampened. Nonetheless, the third type of bowing articulation, ricochet, requires both precession, and the sound loss that comes with it (Beament, 30). Most of the spring in the bow comes from the hair vibration (Beament, 161). This is not possible without precession and the vibration set up along the bow hairs actually aids in an even and measured bounce.

A particular variation of ricochet is the spiccato. Spiccato is a back and forth stroke in which the bow leaves the string. In medium fast spiccato, where the stroke is too fast to place, but too slow to stay on the string, precession aids articulation. In very slow spiccato, it is possible to improve the tone by placing the bow on the string before each note, and orchestral players and quartet players alike prefer this method. (This produces little or no precession). However in medium-fast spiccato, this becomes too difficult, and thus, close proximity to the string is the best chance of eliminating pitch-bending precession. Although precession also plays a small role in sautillé strokes (strokes that sound like the bow is “bouncing” although it never leaves the string) since the bow actually never leaves the string and most of the articulation is due to the changing direction of the string between up and down bow. Thus, both slow spiccato and fast sautillé fall under the category of separate bowing articulation and not under the category of bow precession.



A few additional notes must be made; I list these merely to make the point that a great deal of time must be spent mastering the bow because of its somewhat capricious physics.

1. The bow's physics generally relate to its weight, center of gravity, shape and mass of the head, elasticity, and resistance to permanent or temporary deformation (Peterlongo, 65).
2. The bend of the bow stick also promotes a softened bounce (Johnston, 119). The most important change to the bow between the baroque and the modern bow was that of shape: the concave shape of the modern bow tends to cause the bow to straighten out when the hairs are pulled taut, thus increasing the strength of the bow and the tension on the hair (Johnston, 120).

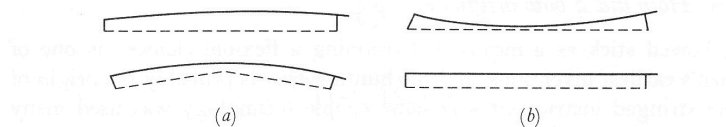


Diagram 20 (Beament, 158).

3. Precession during bouncing strokes can generate longitudinal waves in the bow hairs and cause some instability of tone (Cremer, 120).
4. The tension on the bow hair and wood can greatly interfere with low resonances since the resonances for the bow and wood are within the bandwidth (general area) of the pitches in first position

on the G string. The resulting sympathetic vibration often causes trembling for beginners (Cremer, 123) .

We have said that the violin mimics the voice by its tone—i.e. the formants of the vowels being like the filtering of the violin tone by the violin body—but it also mimics the voice by its transients. In the vocal world, transients are consonants and correspond to partials, although they correspond to a bandwidth of partials and not specific ones (as would vowels). For example, the consonant “P” is a spread of low partials. The consonant “T” has slightly higher partials, and the consonant “K” has even higher partials. With relation to the bow, this simply means that starting close to the fingerboard with moderate bow weight will yield a “P,” starting over the focal point will yield a “T” and close to the bridge will create a “K” sound. (Johnston, 333). This is as far physics research goes, for no one has yet taken the time to measure the transients of the great violinists, although the equipment for this measurement does exist. Without the use of a machine however, we do know that our ears hear literally hundreds of types of transients without being able to explain their differences.

#### Left hand articulation

Just as there are three types of bow articulation, there are three types of left hand articulation. The first is the throwing down of a finger during a slur. When the finger hits the fingerboard, we hear a click, but the largest transient sound actually comes from the unstable vertical and circular vibrations (precession) set up along

the length of the string. The clarity of the transient depends on the strength of these vibrations, which in turn depend on the speed of the finger drop. A landing on the soft part of the finger will round the corner and damp the upper partials of the transient a bit and a landing on the tip of the finger will allow for more of the high overtones to make their appearance (Beament, 21).

The second type of left hand articulation happens when a finger must be pulled off a string during a slur so that a lower pitch may be played. Expert violinists articulate this by pulling the finger off slightly sideways so that it takes the string ever so slightly longer to achieve periodicity at the new pitch. The resulting interval of time initiates the starting transient of the new note. A difficult problem comes about as a result of trills that combine these two first types of articulation. In general, it is advisable to find an angle of the hand and finger that create the most efficient and streamlined motion both of dropping and lifting the finger, so that the finger need not go in a larger motion than necessary. Usually, looking for an oblique angle and dropping the finger on the left side of the string so that it naturally pulls off sideways works well. Since every hand and finger have a different shape, this requires quite a bit of experimentation, but when achieved, creates a perfectly clean and easy-to-execute trill, with only one angle of motion that combines the two types of articulation.

The final type of left hand articulation lies in the shift. In order for a shift to be clean, there must be a moment of release. Logic would tell us that the shorter this release is, the better, but in fact, thinking about short shifts usually leads to a jerking motion which causes the hand to take longer to stabilize on the new pitch.

Let us examine the motions that allow for a clean shift. A shift with the same finger merely achieves a narrow arc in which the pressure on the finger is slightly released during transition to the new note and drops into place on the new pitch creating a small transient. This allows for the release between articulations that we spoke of earlier. However, it is more common to shift on a different finger. When this happens, the best method is to remember that the shortest distance between two points is a straight line and unless a certain type of portamento is desired, the cleanest shift will be one that uses good rhythm (see Appendix A) and allows the original finger to be lifted at the same time and place as the new one is being dropped. The best two-finger shifts approximate the arc of the one-finger shifts. The tips of the fingers move in straight lines that cross each other and the dropped finger creates the new articulation.

I have mentioned that transients help us recognize both different instruments and different players. Even a non-musician would have no difficulty recognizing the difference between the lengthy transient of a brass instrument (which is caused by the large discrepancy in mechanical resistance between the air issuing from the mouth and the air inside the tube) and the generally more crisp articulation of a string instrument (Johnston, 182). Likewise, violinists tell the difference between each other's playing largely based on the transients created by different players. Think about how easy it is to identify a pizzicato transient and a spiccato transient and you will understand how this works (Beament, 127). Transients depend almost entirely on the player, not on the violin and thus, are extremely important in the discussion of advanced technique (Beament, 129).

What follows from the fact that we identify different players by their transients is that most likely, we might sometimes also identify different composer's by the transients. Transients are the single greatest technique of variation available to the violinist. Even tone can only be varied by a certain percentage. The history of conscious thought about transients and their influence in music goes back as far as the baroque period. Every single note of the baroque style was expected not only to have slightly different length, but also a different articulation. The classical period carried this principle along with it, but combined it with a different bow and violin, leading to articulate, crisp transients of increased volume and length and somewhat less capricious in variation between notes. Paganini composed such virtuosic pieces that virtually the only thing that matters is the transients (Soovin Kim). Adjusting the note amplitude after the transient only creates tension in the player's arms. Articulate and release to make room for the next articulation. The romantic period added yet another style of bow with a still slower response and thus less crisp and longer transients, increased importance of the middle of notes, (due to the increased bow curve already mentioned) and increased notation of accents and articulations. The French impressionists certainly had their own style of articulation (or the near absence thereof). The increased and obsessive notation of modern repertoire also gives credence to the notion that transients form an important part of a composer's voice.

Transients are thus more important than we ever dreamed. They clarify phrases, distinguish one note from another, inform us who is playing and may suggest the composer or era of composition. "Character is almost entirely

dependent on articulation.” (Paul Kantor). In short, if you want to be a great violinist, you must master the artistry of articulation.<sup>10</sup>

## **Rhythm**

Rhythm is the great elephant in the room when it comes to professional musicianship. It seems that, first of all, our notation system is flawed; secondly, that when very young, most players develop obsessively bad rhythmic habits. Thirdly, when playing in an ensemble, types of rhythm change drastically. It may seem odd to discuss something quite as abstract as rhythm in a paper on physics, but let me explain why it appears here; the human body forms part of the system of physics that makes the violin, and the human body works in rhythms.

Take the example of walking. It is obvious that extra weight placed on one side of the body by a heavy bag will unbalance the body and create discomfort when walking and severely hamper running ability. What is less obvious is that unequal steps or unnecessarily heavy ones would do the same thing. The constant and steady motion of a balanced body without the interference of conscious thought is nearly perfectly rhythmic. Sometimes, when we start to think about what our bodies are doing, we get worried and suddenly the rhythm is disrupted. Of course even walking tempo changes over time, but it remains so much the same that each

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<sup>10</sup> A side note—continuing sound is of course important at a certain point in composer identification, but since continuing sound is different for different instruments, it does take some time to identify and for some reason unknown to us, is less identifiable to the human ear than transient sound (Beaument, 127).

individual will usually only vary 10 beats each way on a metronome when told to walk at their average pace. Dalcroze began his eurhythmics method with one's natural and unaffected walking tempo.

What most violinists do not realize is that when they create rhythmic instability in their fingers and bow, they not only change the tempos and phrases, but they create an instability of the natural balance of the hand and body. When the body becomes unbalanced, only additional effort can compensate and sometimes this additional effort is not enough to avoid severe technical problems. Fortunately since rhythm, unlike pitch, is sensed in the left side of the brain, it can be approached rationally (mathematically), and practicing with a metronome frequently is a good idea (Beament, 164). Beyond this, the best solution for the violinist is to be aware of the usual tendencies and problems involved with rhythmic accuracy.

Nearly every musician has the same tendencies when it comes to rhythm. You will find a fairly comprehensive list of those tendencies in Appendix 1. The ideas in the list come from Christoph Dohnanyi, long-time conductor of the Cleveland Orchestra, perhaps one of the greatest ensembles in the history of classical music. I cannot overemphasize the importance of recognizing these tendencies. To my knowledge, this is the only list in existence that includes a comprehensive view of Dohnanyi's rules of rhythmic integrity. And a knowledge and application of these principles after practice with a metronome solves nearly all ensemble problems before they happen.

These tendencies are good to be aware of, but they do not tell the whole story. While extreme rhythmic accuracy is important from Beethoven to Bartok, rhythm cannot always be mathematically perfect, or certain types of music lose their lilt: Strauss waltzes and Brahms violin sonatas come to mind. What holds these particular works together if it is not perfect rhythm? It is something far bigger. Perfect pulse. The relative stability of the larger pulses becomes the only firm rhythmic rule in these works, and like the system of rhythm described above, it keeps both the body and the phrase in balance. Awareness of what one is doing is key. When one is aware, choices can become personal. However, without awareness, unfortunate mistakes create difficulty. "Singing and tonguing the rhythm (like a woodwind player) is by far the best way to check if you have figured it out." (Chris Wu) "If you can sing it, you never need to justify your interpretation to anyone." (Paul Kantor)

One more problem of rhythm in ensemble stems from acoustics. The speed of sound is really very slow and any experienced orchestral player knows that sound simply does not get to them at the same time it is played on the other side of the stage (Johnston, 138 check). In addition, energy from a point spreads out spherically (i.e. over a large area) and so at a distance of only 1 meter, the listener only receives about 1/10,000 of the total available energy. (Johnston, 152) Furthermore, according to the "inverse square law" sound level drops with the square of distance. Thus, at two meters (one meter x 2) you only get one-quarter of the sound. (Johnston, 154). Since sound bounces off walls, orchestras usually put a wall behind the players to take advantage of the early reflections which reinforce the direct



sound (Johnston, 154), but of course then you have even more delay time as the sound travels to the wall and then back to you, and the wall absorbs some of the sound (Johnston, 156). All of this means that really excellent orchestral playing or solo playing with orchestra means actually anticipating the other players actions and setting your bow into motion at the exact moment that you expect sound to come forth from their instrument, not the exact moment (a split second later) in which you hear their sound. By then, it is too late and good ensemble is compromised. This goes for chamber ensembles as well, but at least the sound delay is a bit less extreme than in a full symphony orchestra.

### **The Physics of the Body**

The physics of the body could make another entire paper, so I will address it only briefly in relation to the ideas about physics that we have already discussed.

Performers muscles contract when they are performing and thus the blood vessels become smaller. This causes insufficient blood supply and fatigue. Any techniques that can be used to rectify this problem without creating adverse side effects are a good idea (Peterlongo, 81). Here are some of my own thoughts about relaxation with relation to the physics we have already discussed. Since we have already mentioned the importance of rhythm, I will mention again that it is the single most useful tool in balancing and relaxing the muscles of a player. Sometimes, however, an unintentional result comes about as a result of focusing on rhythm or preparing to play. The player spends a moment holding completely still.

This seems like a good idea before beginning a performance or during a rest, except for one thing. The instant that the arm stops moving in mid-air, a certain small amount of tension must be introduced. Although it is small, sometimes this tension is just enough to keep the player from getting their best sound. Veteran orchestral players know this from years of performing excerpts, which require more control and simultaneously more relaxation than any other type of playing. If one is able to move freely in tempo before beginning an excerpt, and during rests in the middle, the chances of success skyrocket. Add to this the concept of efficiency. It will be obvious that excessive movement will unbalance the player's arm, so movement must be efficient and purposeful, always in the direction of the desired motions and in rhythm with the music, no matter how slow, fast, small or large. When deciding on movements, using physics to ones advantage is personal. I will give one example from my own playing. For example, when I start the Mendelssohn Scherzo, I know that my bow must stay connected to the string but sound like it is bouncing. This means I must start from the string with the bow, but sometimes, the note does not speak well, so I add to this the concept of articulation in the left hand and drop my finger right before the beginning of the first note to create a transient that will aid the string in starting its vibrations on time. This creates a gentler but timely transient in the beginning of the soft passage.

Energy transfer is another idea for relaxation. Each time a finger is dropped, energy is given to the sound, and furthermore, because of elastic impedance, to the finger. The natural motion of the finger, were it a hard object, would be to bounce, but since it is soft, the bounce is only slight. I like to use this small rebound as a

slight loosening of pressure either to begin the vibrato on a long note or proceed to the lifting of the finger for the next note in a fast passage. Either way, the finger must never be held down or remain immobile with the same force used to drop the finger, for this creates tension. The hardest part of the motion, dropping the finger, has already been completed and inertia will keep the string in place even if nearly all of the weight on the finger is released. (Of course, the thicker the string, the more a certain amount of weight must remain.) The important fact is that this weight need only be the minimum for producing a good sound, and constant motion of the hand, either by lifting and dropping fingers in a fast passage, or vibrato in a slow or medium one, will give the violinist the necessary relaxation to play accurately and comfortably.

The last item we will discuss is one of perception, considering both the human ear and the physical property of intensity. Here we turn to two of the senses of the body, namely hearing and touch. As do all the five senses of the body, hearing and touch operate logarithmically, which means that the *amount* of change varies as the base values vary. (Johnston, 239). To clarify this, consider that in the case of touch, the fingers are sensitive to a percentage of pressure; the ear is sensitive to a percentage of loudness. However as the sound pressure level is reduced well below the usual speaking voice level, the ear becomes more and more sensitive to level changes. Of course this means that pianissimos are harder to maintain than fortissimos because with the smaller total amplitude of a pianissimo, even a slight change is enough for the ear to perceive a significant percentage difference

(Beament, 130). This is the reason that orchestras ask for pianissimo excerpts; the judges can hear more of the variation in loudness.<sup>11</sup>

A similar thing happens with hearing. The ear recognizes pitch by a long row of tiny resonators. As the pitch increases in frequency, the number of resonators responding to each vibration per second decreases. This causes the greater difficulty of hearing perfectly in tune when playing extremely high on the E string (Johnston, 234). When a pitch sounds, a band of resonators which corresponds with that pitch begins to vibrate, and bands of resonators corresponding with the partials also vibrate. This is called bandwidth resonance. In general, the louder the note, the greater the number of vibrating resonators. The perception of loudness depends on the number of nerve cells firing, and again this increases logarithmically. Therefore, loudness can be increased either by increasing the volume of the fundamental or the partials (Johnston, 239). The ear has its greatest response around 4000 Hz because this is the resonant frequency of the ear canal (Johnston, 234). So then, although the ear is most sensitive to the higher notes of the violin, the lower notes have far more audible harmonic content and this helps to balance things a little. In the end

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<sup>11</sup> Power also factors into aural perception. Power is proportional to the square of the sound pressure and thus increases by four when the sound pressure is doubled (PL 34). Sound power must be approximately tripled (10 Db) to produce an aural perception of only double the perceived loudness (VE 5). Also, the ear requires much higher sound levels in the bass to match the perceived loudness levels of the higher frequencies. Let us take the example of the Mendelssohn Midsummer Night's Dream Scherzo again. In order to maintain apparently equal dynamics in vastly different ranges, the player must put a great deal more energy when transferring to a lower string. Only through this unequal treatment of the various pitches will the sound become apparently equal to the listener. As dynamic level increases, the Fletcher-Munson curve tells us that intensity balances out more easily.

however, the violinist must compensate in the low register because of the ear's sensitivity in the high register (Peterlongo, 38; Peterlongo, 100-101).

### **Intonation**

The final question of the ear which we will deal with concerns what Johnston calls "the puzzle of consonance." (Johnston, 240). I must state that the best research has only succeeded in suggesting theories. No conclusive results have been reached, but I think some of these theories have promise. Experiments using pure tones have helped physicists to discover that even without partials, pitches closer than a minor third vibrate not only their respective resonators, but cause the bandwidth resonators in the ear to overlap because they are too close to each other (Johnston, 236). It can be easily seen how this would cause dissonance. The vibrations clash because they are close to each other in the ear and cause interference with each other (Johnston 240). But then, why do we hear a Major 7<sup>th</sup> as dissonant? Clearly something else has come into play. This brings us to perhaps the largest can of worms we have to open: intonation.

Intonation is an endless topic. Even the very best violinists have different ideas about pitch relationships. For perhaps the best discussion of this, I refer the reader to an excellent book called *"How Equal Temperament Ruined Harmony (and Why You Should Care)." In spite of the many differences of opinion among professional musicians, there are some important general principles to discuss.*

First of all, intonation relates directly to partials. You will recall that partials always vibrate at a frequency that is a whole number multiple of the fundamental. ( $F \times 1$  (octave),  $F \times 2$  (5<sup>th</sup>),  $F \times 3$  (15va),  $F \times 4$  (Maj. 3rd), etc. (Johnston, 19). One way of thinking about consonant frequencies is to use pendulums to describe the theory of consonance. If one imagines a pendulum swinging slowly at the fundamental pitch, and two pendulums representing the first two overtones swinging twice and three times as fast respectively, you will realize that the swings line up on a regular basis. Since the “swings” line up at regular intervals we perceive these partials and the fundamental pitches that coincide with them (at least the lower numbered ones) as consonant (Johnston, 32). The more frequently the cycles line up, the more consonant we perceive the pitches.

We have learned a little about the human ear and how it responds to vibration. In a musical note,  $F \times 2$  (or the first overtone) is always ringing along with the fundamental, so the ear perceives vibration in more than one location. Since  $F \times 2$  is exactly double the frequency of the fundamental, on the violin, if one plays an octave double stop, the higher pitch should ring exactly in tune with the second partial of the lower pitch. Otherwise, the ear will perceive higher pitch as dissonance.<sup>12</sup> If the major 7<sup>th</sup> we mentioned in the previous paragraph is sounded along with the lower note, it will certainly be perceived as dissonance, not due to its distance from the fundamental of the first note, but from its proximity to the  $F \times 2$  of

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<sup>12</sup> The of using sympathetic resonance to hide minor problems of intonation (a form of “cheating,” if you will) has been discussed earlier in this paper. However, this will only cover over very minor errors in intonation; more than that will be perceived as a dissonance. Furthermore, a properly played octave will have much more of a “ringing” sound, owing to the reinforcement of partials.

that tone. This solves the question of consonance. The consonance and dissonance of chords falls along the same line. Let us look at a major 3<sup>rd</sup>. See how the partials of the first note line up with the partials of the major 3<sup>rd</sup> above it. A perfect fifth is even more consonant. Thus, a major triad is an extremely consonant chord (Johnston, 242-243). Bold lines represent overlapping partials. (Johnston, 103). (See Diagram 21.)

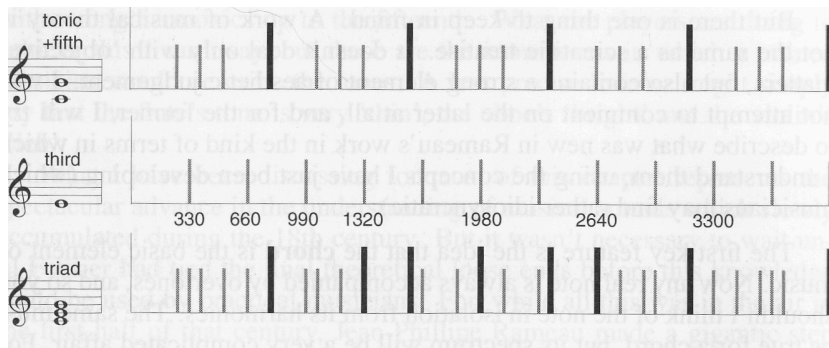


Diagram 21 (Johnston, 101-102).

Minor thirds are thought to be perceived as less consonant than major 3<sup>rds</sup> simply because the minor third does not occur as early in the harmonic series and the fifth partial (major 3<sup>rd</sup>) creates a dissonant half-step with the minor third. This is quite possibly why we perceive the minor triad as less jovial than the major one. But what I have not mentioned is that these “acoustically pure” thirds are VERY different from what most violinists believe.

Major thirds that line up with overtones (i.e. acoustically pure) will actually be lower than most musicians would tolerate. The ratio of the acoustically pure

interval is  $5/4$  and this is several cents lower than the major third in equal temperament. Minor thirds correspond to the ratio of  $6/5$  and will in contrast be extraordinarily higher than equal temperament tuning. And although a perfectly tuned third may sound good in a chord, it will sound so unusual in a scale that it may disrupt a melodic line (Duffin, 72). Therefore, a pure third can be (and probably should be) used at a point of arrival or in a chord, but otherwise, we must find another solution (Duffin, 71). This brings us to our next topic.

Most musicians are familiar with the concept of temperament. In the strictest sense, violinists do not have temperament because most of their pitches are not fixed. However, since we spend so much time playing with instruments such as the piano, that do have temperament, it is important for us to know about it. Perfectly equal temperament (the perfectly equal division of 12 notes to the octave) does not actually exist in the music world. Even pianos do not follow the model of “pure” equal temperament. Rather, they follow Bach’s model of “well temperament”, which means that a compromise—usually Vallotti temperament or some version of meantone, not equal temperament—has been reached to facilitate performance in different keys (Johnston, 40).<sup>13</sup> Without discussing further all of the different types of temperament, I will mention only that a complete circle of absolutely perfect fifths (ones that match up with overtones) will gradually cause the pitch to creep up so far that the corresponding octaves will not line up.

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<sup>13</sup> It is true that with Vallotti and meantone temperaments, not all keys will sound the same. Each one will still have a different “flavor,” but none will be unusable.



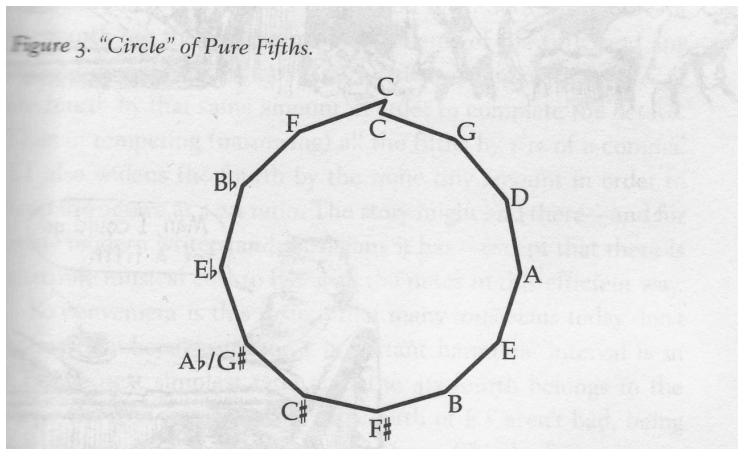


Diagram 22 (Duffin, 25).

This is the root of intonation dilemmas. One must match the partials, but if the partials are matched in every situation, the pitch must constantly change when alternating suddenly between the high and the low registers, and this in turn sounds out of tune. This is why the varying Baroque temperaments were created. While it is probable that Schubert and Brahms, being primarily pianists, thought in equal temperament (as evidenced by certain enharmonic spellings; Duffin, 85), Russell makes the assertion that Mozart, Bach and Beethoven all thought about pitch in terms of sixth-comma meantone, or 55 divisions to the octave with two sizes of half step (Duffin, 51, 53, 55, 57, 74, 86). I concede that his arguments are good and this is both possible and probable.

This leads us to ask: How should a violinist approach intonation since temperament is not fixed? There are generally two schools of thought. One is called just intonation and relates to perfect interval ratios (Johnston, 19). Although ideal, it is not completely possible in modulating music. The other is called expressive

intonation and usually refers to bending pitches towards the note to which they are headed (Russell, 19). This causes pretty severe problems with chordal intonation. Just intonation is not always possible, as has already been mentioned, due both to the creeping fifths and impossibility of consistently perfect thirds. However, if one agrees to compromise on thirds and sixths a bit, it becomes possible to tune the octaves and fifths perfectly. Melodic intonation is an excellent plan when one is playing single pitches, but as soon as they have to line up with a chord, it becomes problematic. Clearly the only good solutions come on a case-by-case basis.

I humbly submit that intonation is a personal decision, but there are some guidelines that may be helpful. There is one thing that all musicians agree on. Octaves must be perfectly matched to the partials. Beyond that, there are places where vertical intonation must take precedence and others in which melodic intonation must take precedence. Recordings by Joachim, Sarasate, Ysaye, and Remenyi all reveal very different and personal major scales (Duffin, 126), and we learn therefore, that more than one solution is possible. Arnold Steinhardt describes quartet playing as a compromise of just, mean tone, expressive, and equal temperament intonation (Duffin, 69). What matters most is awareness of what one is doing. Then the player can decide upon a system with relevance to the music and the overtones present. Beyond this, a player should of course adhere to his own system throughout a work.

In the end, an understanding of physics makes us more keenly aware of our own personal and artistic choices and aids us in developing a more thorough understanding of our instrument. And there we must end our discussion of physics

and the violin: with many questions left unanswered. In my humble opinion, this is most appropriate, for as all musicians know, science has a limit. A violinist creates beauty not with mathematical equations, but rather the feelings and subtle sensations of the hands and the imagination.

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## Appendix 1: Rhythmic Tendencies

### (The Rhythmic Conduct Code of the Cleveland Orchestra)

| Tendencies  | Solutions  |
|---|--|
| Poor section ensemble at entrances  | Start every note from the string   |
| Rushing during long passages of fast notes  | Practice with metronome from day 1. Fill out beat.   |
| In $\frac{3}{4}$ , 3 <sup>rd</sup> beat is always late and long.  | Practice with metronome, subdivide   |
| In 6/8, 3 <sup>rd</sup> beat and 6 <sup>th</sup> beat are always late and long  | Practice with metronome, subdivide   |
| Insecure cut-offs for long notes with rests afterwards  | Hold long note and subdivide until the exact moment in which the rest enters.  |
| Long rests and notes tend to get shortened  | Listen to other parts and subdivide  |
| Lack of difference between dotted eighth/sixteenth rhythms and triplet rhythms without middle note  | Subdivide  |
| Pizzicatos rush   | Metronome-take time for full sound.  |
| Entrances after ties or downbeat rests are ALWAYS late and lack rhythmic integrity.   | Subdivide and anticipate beats. Come off tie on time and “play slowly.” There is no need to rush to make it to the next beat if you come off the tie on time. Fill up beat with the notes that are written. Practice with metronome. |
| In mixed rhythms (rhythms combining more than one kind of note, such as “quarter, eighth, eighth,” or “eighth sixteenth, sixteenth”) fast notes tend to be compressed and slow notes tend to drag. When these figures alternate between parts, often two different tempos exist simultaneously. | Take time to sing shorter note values. Play all notes in the same tempo from using a metronome during ALL practice. Subdivide always.  |

### **Annotated Bibliography for the Physics of the Violin**

**Beament, James. *The Violin Explained*. Oxford: Clarendon Press, 1997.**

Beament writes for performers. His book, *The Violin Explained*, is perhaps the most accessible source on the specific physics of the violin. However, while many of the overall principles are addressed accurately, *The Violin Explained* lacks sufficient detail for a work of its length. In addition, the author often states the principles of physics without application to the player. This causes the source to appear somewhat undefined in its purpose. Beament's experience as a professional violinist makes his descriptions of technique accurate and valuable.

**Cremer, Lothar. *The Physics of the Violin*. Trans. John S. Allen. Cambridge: The MIT Press, 1984.**

Cremer gives the complete mathematical formulas for practically every known physics principle influencing the violin in this authoritative work of nearly five hundred pages. Although originally written in German, it has been translated to English. Cremer wrote the book for his physics students and colleagues and it consists in large part of equations and mathematical proofs. A thorough understanding of the basic physics of the violin is an assumed pre-requisite.

**Duffin, Ross W. *How Equal Temperament Ruined Harmony (and Why You Should Care)*. New York: W.W. Norton & Company, 2007.**

This intensely interesting work discusses not only the physics of partials, but also their relation to temperament and intonation. Duffin devotes a chapter to discuss the physics of pitch for the violin and other non-tempered instruments. He writes about the relation of intonation to key, its complications in quickly modulating music, and its difficulty in atonal compositions. This recently published source draws both from the experience of professional musicians and from physicists.

**Galamian, Ivan. *Principles of Violin Playing and Teaching*. Englewood, New Jersey, Prentis Hall, Inc., 1985.**

Galamian's book on violin playing has become one of the most revered treatises on teaching the violin. His clear, direct approach to basic and advanced technique sets his book apart from other treatises, which are notably less organized and usually somewhat convoluted. His topics include everything from the various approaches required for different shapes of hands to the execution of advanced bow strokes such as sautillé and ricochet.

**Hall, Donald E. *Musical Acoustics*. Sacramento: Brooks/Cole, 2002.**

Musical Acoustics is one of the most commonly used college textbooks for students who are studying acoustical physics. I have used this source as an authoritative verification for information in some of the sources written for musicians. While Hall devotes little time to the violin specifically, he does devote a reasonable amount of time to the general principles of stretched strings. The principles from this book aid the development of an overall understanding of music physics. The information is more thorough than the information in *Measured Tones*.

**Helmholtz, Hermann. *On The Sensations of Tone*. Trans., Ellis. New York: Dover Publications, 1954.**

The famous work by Helmholtz forms one of the first and most foundational discussions of physics and sound. Helmholtz is almost singlehandedly responsible for developing the field of musical acoustics and much of his research still serves as the basis for sound engineering and acoustics today. Perhaps one of his most valuable contributions was his theory concerning the vibrations of stretched strings.

**Johnston, Ian. *Measured Tones*. Philadelphia: Institute of Physics Publishing, 2005.**

Ian Johnston's *Measured Tones* is a brilliant work about basic musical acoustics. Topics cover the practical acoustics of all instruments including the voice, and Johnston devotes a great deal of time to string instruments. He possesses an unparalleled ability to explain complex concepts in their simplest terms. In an effort to make his writing accessible, Johnston sometimes oversimplifies, but the book still makes an excellent entrance point for professional musicians who have little knowledge of physics.

**Knill, Oliver. *Linear Algebra and Differential Equations*. Harvard, Fall 2003. Web. 20 January 2014.**

This webpage details the equations for the resonant frequency of the Tacoma Narrows Bridge. The webpage originates from a Harvard math course.

**Peterlongo, Paolo. *The Violin: Its Physical and Acoustic Principles*. New York: Taplinger Publishing Company, 1979.**

Peterlongo introduces his readers to the history of violin making in his work on violin physics. While he does discuss basic physics principles, perhaps the most useful contribution of this source lies in its analysis of older instruments. Peterlongo is able to point to specific acoustic principles that cause the varying characteristics of the old Italian instruments, even comparing Stradivarius' and Guarnerius' violins. Peterlongo also compares these older instruments to modern violins and notes the effect of the physical differences on the sound.